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DECEMBER, 1923

The Effect of the Material Composing the Sides of Deep Slits on the Intrinsic Intensity of Light Transmitted Through the Slits 1069
CLARENCE R. SMITH

On the Relation between Time and Intensity in Photographic Exposure 1079
LOYD A. JONES and EMERY HUSE

Color Match and Spectral Distribution 1115
W. E. FORSYTHE

Studies Based on the Spectral Complementaries 1123
R. H. SINDEN

INSTRUMENT SECTION

An Instrument for Showing Some of the Phenomena of Radioactivity 1155
J. E. UNDERWOOD

New Procedure in Tests of Auditory Acuity 1161
C. C. BUNCH

An Automatic Focusing Device for Photomicrography with Ultraviolet Light or Light of Any Wave Length 1167
W. T. BOVIE and C. E. BARR

The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion 1175
IRWIN G. PRIEST

Diffusion of Light from a Searchlight Beam 1211
ENOCH KARRER and U. M. SMITH

Methods of Making Temperature Corrections in Direct Reading Potentiometers for Measuring Hydrogen-Ion Concentrations 1235
W. T. BOVIE

The Measurement of Ionization Currents 1245
ROY KEGERREIS

Reviews 1113, 1251-6
Notes 1078, 1121, 1154, 1159, 1160, 1174, 1210
Index to Advertisers XXXV

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Journal
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Vol. 7 DECEMBER, 1923 Number 12

THE EFFECT OF THE MATERIAL COMPOSING THE SIDES OF DEEP SLITS ON THE INTRINSIC INTENSITY OF LIGHT TRANSMITTED THROUGH THE SLITS

By Clarence R. Smith

Introduction

The phenomena occurring when light is passed through a narrow, deep slit appear to be very interesting and more complicated than one might at first suppose. There is not only strong polarization but also marked absorption. Moreover this absorption bears an interesting relationship to the wave length of the light, the depth of the slit, the width of the slit and also the material of which the walls of the slit are composed.

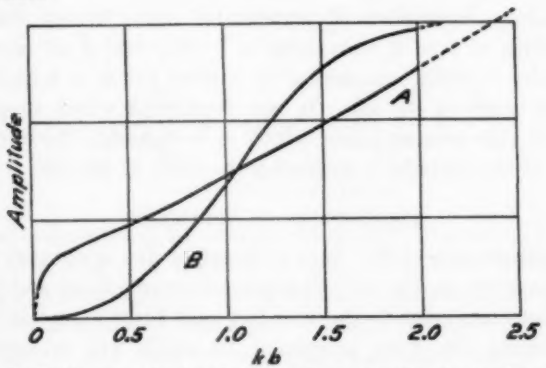

Fig. 1. *Rayleigh's curves showing the amplitude as a function of a number proportional to the width of the slit. A—electric vector perpendicular to direction of slit. B—electric vector parallel to direction of slit.* $k=2\pi/\lambda$. $b=\frac{1}{2}$ *width of slit.*

Lord Rayleigh[1] has treated theoretically the polarization of light by a narrow slit though he assumed the slit to be in an infinitely thin

[1] Lord Rayleigh. Roy. Soc. Lond. Proc. A, *89*, p. 194; 1913–14.

screen. Nevertheless his curves (Fig. 1) show a close resemblance in general shape to the experimental curves obtained by Sieg and Fant[2] (Fig. 2) with "deep" slits, and also to the curves shown in this paper.

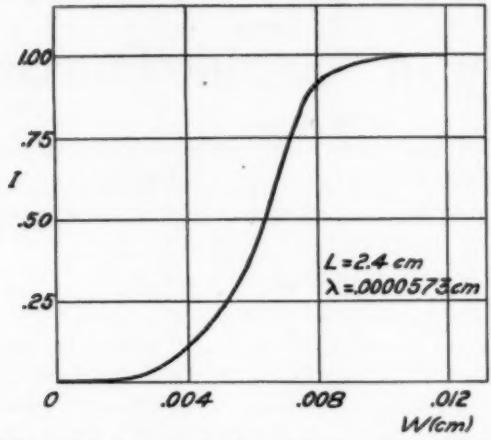

FIG. 2. *One of Sieg and Fant's curves showing the intensity as a function of the width of slit.*

Stewart[3] in some experiments performed in another connection noticed that "a narrow slit, in a thick screen, can . . . by being made sufficiently narrow, extinguish all transmitted light, though the slit still have a width of 5 to 8 wave lengths." Sieg and Fant[4] investigated the intensity of light transmitted by a deep slit as a function of the width and depth of the slit. It was their work which suggested the problem of this present paper which is to consider the effect of the variation of the material constituting the walls of the slit.

DESCRIPTION OF APPARATUS

The arrangement of the various parts of the apparatus is shown diagrammatically in Fig. 3. The general arrangement and principles of operation are similar to those of Sieg and Fant[5] except in regard to several details, the more prominent of which will subsequently be pointed out.

From the slit S_1, of a monochromatic illuminator, light was received by the lens L_1 and sent forward in a parallel beam. The lower portion

[2] L. P. Sieg and A. T. Fant. J. Op. Soc. Am., *5*, p. 218; 1921.
[3] G. W. Stewart. Abstract. Phys. Rev. *5*, p. 73; 1915.
[4] Sieg and Fant. loc. cit.
[5] Sieg and Fant. loc. cit.

of this beam was interrupted by mirror M_1 and projected over to mirror M_2 while the upper portion passed on to the slit S_2. The horizontal position of the edge of the mirror was considered to be better than the vertical position used by Sieg and Fant not only because of greater ease in adjustment but also in offering less opportunity for slight differences in wave length between the two divided portions of the light beam.

The light passing over the upper edge of M_1 reached S_2 which was the slit under investigation. Between S_2 and P only one lens was used instead of two as in Sieg and Fant's arrangement, and the light

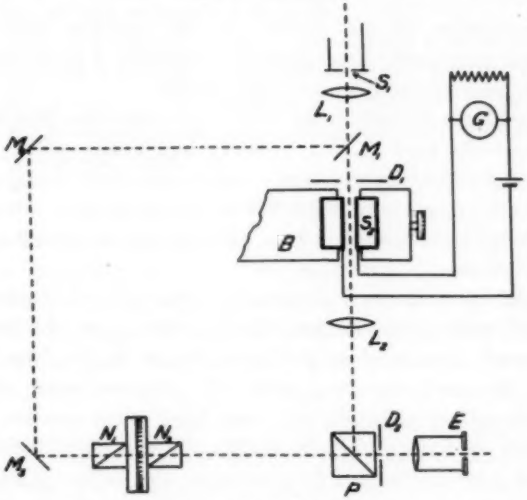

FIG. 3. *Diagram of Apparatus.*

instead of being sharply focused upon the silver strips in P, was slightly diffused into a spot of appreciable size. This was thought to be of advantage since the eye is more reliable in matching areas of larger size and lower intensity than areas of smaller size and higher intensity.

The cube P was made of two right-angled glass prisms with silver strips cemented between. The use of the diaphragms D_1 and D_2, and also the mirrors M_2 and M_3 will be readily understood from the diagram. One of the blocks used to form the slit was rigidly attached to the bed of a Michelson interferometer B; the other was set in wax on the movable carriage of the interferometer. The galvanometer G was used to register contact between the blocks, or zero width of the slit.

By rotating the nicol N_2 with reference to the stationary nicol N_1, the intensities of the two images in P could be made to match when viewed by means of the eyepiece E.

For the measurements made with steel, the slit was formed by two of the plane surfaces of two similar blocks which had been made by cutting in two a Hoke Precision Gauge. Thus the original surface of the gauge was used. For the other metals employed, the surfaces were especially prepared by the method described below. The depth of the slit in every case was 2.31 cm.

EXPERIMENTAL

The preparation of the various metallic surfaces composing the walls of the slits under investigation comprised a prominent feature of the work and will be described specifically.

First two blocks of steel about $10\times75\times75$ mm were ground together with successively finer grades of emery and finally with rouge until the surfaces gave interference bands which were fairly straight. Finally the surfaces were carefully inspected and from the best areas were cut smaller blocks each having a part of the original surface measuring about 7×23 mm.

Four pairs of these small blocks were selected and the flat surfaces of each pair then plated electrolytically with one of the four metals, nickel, copper, silver and gold. In each case it was found that the flatness of the new surface was fairly well preserved after this deposit. A slight amount of polishing was done upon these new surfaces with French chalk on a piece of chamois skin. It was found, however, that this sort of polishing tended to wear away the surface faster near the edges and thus destroy the flatness, hence it had to be discontinued before a very highly reflecting surface was obtained. The nickel came from the plating bath with the brightest surface of any of the four metals tried while the silver had the dull appearance which is characteristic of electrolytic silver. Enough polishing was done upon the silver surface to render what might be termed a bright appearance though not a brilliant mirror. When in the condition finally used all the surfaces were flat to within about 0.0001 cm.

With a given pair of surfaces set face to face, the slit was adjusted for a wide opening and then the light transmitted was matched in P with that coming by way of the path through the nicols when N_2 was set for a certain position chosen as a maximum. This was accomplished by means of a neutral tint photographic wedge placed before S_2.

The actual maximum position of 90° for N_2 was not used but rather an angle of about 60° as observations were more easily made near this angle.

Starting with the slit at zero opening it was gradually widened at intervals of 0.001 cm. Three independent adjustments of N_2 were made for each value of W thus chosen. These were recorded as 1st, 2nd, and 3rd trial as illustrated in Table 1.

The intensity I for any position is given by

$$I = k \sin^2\theta \tag{1}$$

where θ is the angle through which N_2 has been rotated from the dark position and k is a constant so chosen that for the width at which the intensity became constant, I was unity.

In the table one typical series of observations is given as a sample. In Figures 5, 6 and 7 are compared the results of the five metals used for each wave length λ. The depth of slit L was 2.31 cm in every case. Throughout the work it is intrinsic intensities which are obtained and not total transmitted light.

CONCLUSIONS

Sieg and Fant[6] have given a theoretical discussion of the relationship between the intensity of light transmitted by a slit and the width and depth of the slit. For the diffracted light within any angle θ, they give the following expression for the resultant intensity of light passing through a slit of given depth, assuming unit intensity for a slit which is infinitely thin:

$$I = \frac{1}{0.45\pi}\cdot\frac{\pi}{m}\cdot\frac{1}{2}\left[\sum_{i=0}^{i=m}\frac{\sin^2 a_i}{a_i^2}R_s^i + \sum_{i=0}^{i=m}\frac{\sin^2 a_i}{a_i^2}R_p^i\right] \tag{2}$$

In this expression a is an auxiliary angle defined by the relation

$$a = \frac{\pi w}{\lambda}\sin\theta$$

where w is the width of the slit and λ the wave length of the light. In (2) a_i represents the value of a for i reflections, and m the number of reflections when $a = \pi$. R_s and R_p represent the reflection constants for the electric vector parallel and perpendicular respectively to the plane of incidence. R_s and R_p are assumed in the above expression to be constant, though as pointed out by the authors themselves, the R's are in reality not constant but are complicated functions of the angle a. A more rigorous expression than (2) might be formulated as a definite integral but its evaluation would be well-nigh impossible owing to the complicated relations between the R's and a.

[6] Sieg and Fant. loc. cit., p. 221.

TABLE 1. *One Series of Observations Shown as a Sample*

L=2.31 cm λ=.0000650 cm Material=copper

W cm	Scale readings for θ				θ deg.	$\sin^2\theta$	$I = k \sin^2\theta$
	1st trial	2nd trial	3rd trial	Av.			
.00000	329.9	332.0	331.8	331.2	0.0	.000	.000
.00100	327.4	328.3	327.7	327.8	3.4	.004	.006
.00200	326.0	325.9	326.6	326.2	5.0	.008	.013
.00300	324.8	324.8	325.0	324.9	6.3	.012	.019
.00400	321.1	321.0	321.7	321.3	9.9	.030	.048
.00500	318.7	319.1	320.9	319.6	11.6	.040	.064
.00600	313.7	311.9	312.4	312.7	18.5	.101	.163
.00700	313.8	312.9	312.7	313.1	18.1	.097	.156
.00800	314.4	313.8	314.0	314.1	17.1	.086	.138
.00900	305.3	306.8	304.8	305.6	25.6	.187	.301
.01000	300.7	301.8	302.3	301.6	29.6	.244	.391
.01100	297.5	300.9	297.5	298.6	32.6	.290	.467
.01200	301.6	300.7	301.0	301.1	30.1	.252	.405
.01300	305.7	301.2	300.9	302.6	28.6	.229	.369
.01400	300.8	300.4	303.0	301.4	29.8	.247	.398
.01500	299.8	300.3	299.6	299.9	31.3	.270	.435
.01600	299.6	301.8	301.2	300.9	30.3	.255	.410
.01700	294.9	298.4	297.3	296.9	34.3	.318	.512
.01800	297.0	299.4	299.0	298.4	32.8	.294	.474
.01900	295.2	295.7	299.0	296.6	34.6	.322	.518
.02000	294.6	297.9	296.4	296.3	34.9	.327	.526
.02100	294.6	294.9	296.3	295.3	35.9	.344	.554
.02200	292.6	290.6	289.5	290.9	40.3	.418	.674
.02300	292.6	293.5	290.6	292.2	39.0	.396	.638
.02400	289.3	289.2	289.5	289.3	41.9	.446	.718
.02500	291.0	288.5	287.9	289.1	42.1	.450	.725
.02600	286.9	289.0	287.9	287.9	43.3	.470	.757
.02700	285.5	285.5	285.9	285.6	45.6	.511	.823
.02800	285.0	283.8	285.0	284.6	46.6	.528	.850
.02900	284.0	282.3	286.5	284.3	46.9	.533	.858
.03000	283.9	285.6	282.5	284.0	47.2	.538	.867
.03100	283.0	283.9	284.3	283.7	47.5	.544	875
.03200	285.8	277.5	279.2	280.8	50.4	.594	.956
.03300	283.5	284.1	280.3	282.6	48.6	.563	.907
.03400	280.0	283.4	284.6	282.7	48.5	.561	.903
.03500	279.9	281.2	279.0	280.0	51.2	.607	.977
.03600	278.3	285.7	283.0	282.3	48.9	.568	.915
.03700	277.5	278.0	283.0	279.5	51.7	.616	.992
.03800	277.9	280.8	281.0	279.9	51.3	.609	.981
.03900	283.3	278.0	276.4	279.2	52.0	.621	1.000

That the reflection constants play a prominent part in such an expression as the above is evidenced experimentally by the results of the present work, in which for the same values of λ, L and W,

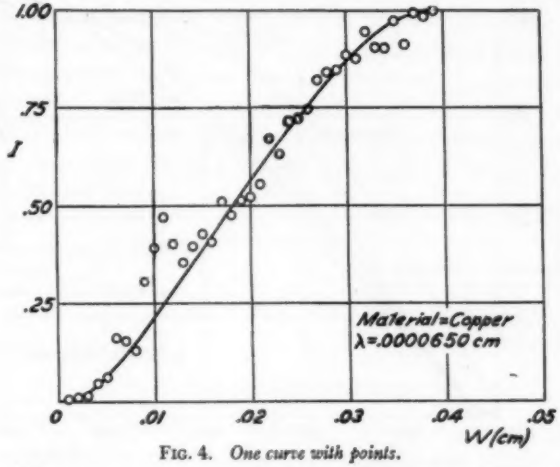

FIG. 4. *One curve with points.*

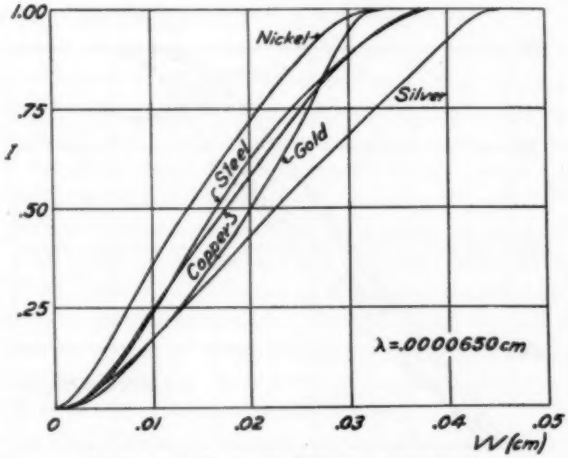

FIG. 5. *Comparison of results for* $\lambda = .0000650$ *cm.*

different materials were used to make up the walls of the slit. The results obtained are somewhat irregular but a comparison of the curves shows at once that differences in intensity of the light transmitted by a deep slit, are produced by changing the material though the other conditions mentioned above are kept constant.

It will be noticed that the curves for the various metals agree fairly well for small and for large openings of the slit while the greatest differences occur in an intermediate region. For large openings a

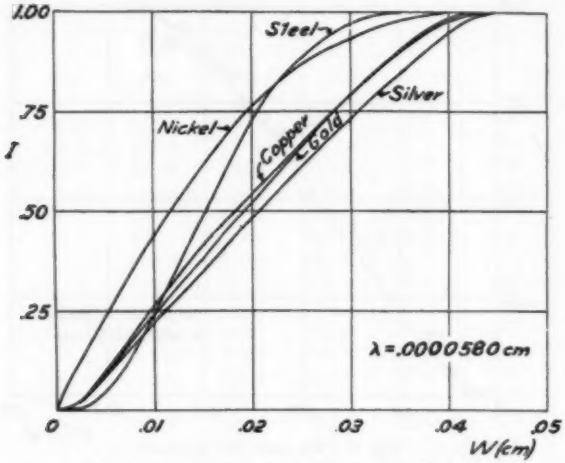

FIG. 6. *Comparison of results for* $\lambda = .0000580$ *cm.*

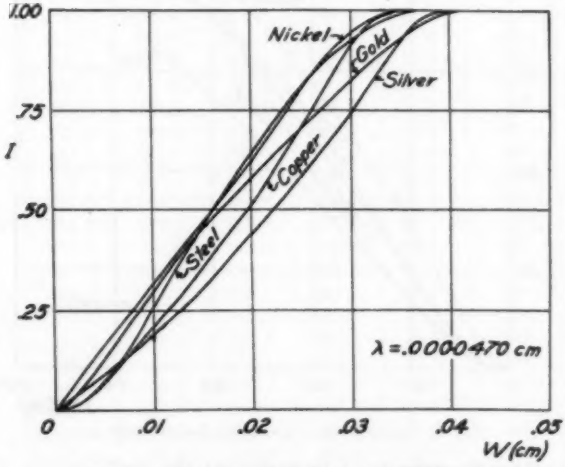

FIG. 7. *Comparison of results for* $\lambda = .0000470$ *cm.*

greater portion of the light is directly transmitted and not subject to reflection at the walls of the slit, hence would not be influenced by the material and we should expect the curves to be similar. This

situation is represented in Fig. 8 (a) where *o* is any point on the incident wave front *ab*. However as the opening is made less, the light which is directly transmitted becomes less in proportion to that which is diffracted and reflected as shown in Fig. 8 (b). Here the material composing the sides of the slit will play a more prominent part and greater differences in the curves would be expected. As the slit becomes still narrower a relatively greater portion of the transmitted light will have been subject, not merely to one or a few reflections, but many reflections from side to side before emergence. Since absorption

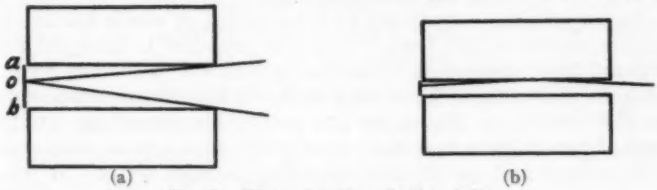

FIG. 8. *Illustrating effect of width of slit.*

occurs at each reflection, the intensity of the light will gradually approach zero for zero width of the slit. Hence the curves approach each other near the zero point.

The probable error was calculated for several values of I chosen at equal intervals from a typical series of data. The results obtained showed the probable error to average about 5%. The chief source of error throughout the work is in the inability of the eye to match accurately the light areas. Compared with this, other sources of error involved are considered to be insignificant.

In the present work no account has been taken of the condition of the light with respect to polarization. The light incident upon the slit under investigation was doubtless slightly polarized by the slits of the monochromatic illuminator. Further work should be done to determine the effect of using completely polarized incident light and varying the direction of the electric vector from parallel to perpendicular with respect to the plane of incidence.

The writer wishes in conclusion to acknowledge his indebtedness to the staff of the Physics Laboratory of the University of Iowa and especially to Prof. L. P. Sieg, who suggested the problem, and whose advice and continued interest were of invaluable aid throughout the work.

UNIVERSITY OF IOWA,
IOWA CITY, IOWA,
JULY, 1923.

Influence of Electric Field on the Spectrum of Helium.—Tschulanowsky suggests that certain lines which appear in the emission spectrum of helium when an electric field is applied to the gas are produced by electron transitions in which the final orbit is characterized by one or two azimuthal quanta, and the initial orbit by four, five or more azimuthal quanta, transitions contrary to the selection principle and therefore occurring only in an electric field or under other abnormal circumstances. Liebert, for example, found that the individual lines of the diffuse series of doublets, designated by $(2p-md)$, were accompanied by $(m-3)$ additional lines in the presence of electric field ($m=3, 4 \ldots 9$ in the observed cases). The lines $(2p-md)$ which are always present, correspond to a transition in which the azimuthal quantum number k falls from 3 to 2; it is proposed to interpret the nth additional line as due to a transition in which k falls from $3+n$ to 2. Analogous observations have been made on the diffuse series of simple lines $(2P-mD)$; on the series $(2S-mD)$, corresponding itself to a change of two units in k, so that none of the lines appear unless electric field is acting; and on the corresponding doublet series. It has also been observed that the additional lines near those of the series $(2P-mD)$ do not approach the series lines as the field is reduced to zero, but approach frequencies separated from those of the series lines by a finite interval. The author has measured some of these lines on plates of Paschen's, and compared their frequencies with those calculated from the energy associated with the orbit characterized by four azimuthal quanta, as deduced from observation on the Bergmann series in the infrared. The agreement is not very satisfactory and much further work should be done to test the theory. [W. Tschulanowsky (Tübingen) ZS. f. Physik *16*, pp. 300–313, 1923.]

K. K. DARROW

Attempt to Determine Electron-orbits in the "Excited" Helium Atom by Quantum-conditions.—Born and Heisenberg apply the methods of the problem of three bodies to attack the problem of the helium atom, simplified by the condition that one electron shall be much further away from the nucleus than the other, under which it is permissible to treat it as a problem in perturbed orbital motion. The approximation should be adequate for the orbits of higher quantum-numbers. After a very elaborate analysis they conclude that agreement with experiment definitely cannot be attained in this way, and that either the motion of the electrons in the stationary states does not conform to classical mechanical laws, or the usual quantum conditions are not applicable. [M. Born (Göttingen) and W. Heisenberg (Munich) ZS. f. Phys. *16*, pp. 229–243; 1923.]

K. K. DARROW

ON THE RELATION BETWEEN TIME AND INTENSITY IN PHOTOGRAPHIC EXPOSURE*

By Loyd A. Jones and Emery Huse

A relation between the intensity of radiation causing a photochemical reaction and the magnitude of the photochemical change produced was found by Bunsen and Roscoe (1) in their classical researches. It was definitely established by them that the effect produced is proportional to the product of the intensity by the time during which the radiation is permitted to act and furthermore, that this effect is constant so long as the product of the intensity by the time is constant regardless of the absolute magnitude of these factors. In other words, if the exposure or insolation, as it was called by Bunsen and Roscoe, is represented by E, the intensity of the acting radiation by I and the time during which this radiation acts by t, the law may be stated by $E=I\cdot t$. According to this law, commonly referred to as the reciprocity law, it makes no difference whether I be large and t small or vice versa so long as the product is constant. For many years it was thought that this law applied rigidly to the case of light sensitive photographic materials.

Scheiner (2) found in connection with the photographic photometry of stars that this relation did not hold. For instance, he found that increasing the exposure in stellar photography 2.5 times, which should be equivalent to an increase of one magnitude in star brightness, corresponded only to an increase of about .7 magnitude. These experiments, therefore, caused the validity of the reciprocity law to be questioned and a year or so later Abney (3) definitely proved by sensitometric measurements that the reciprocity law was not valid.

Since that time this conclusion has been confirmed by many observers and there is no doubt that the Bunsen-Roscoe law fails to agree with experimental observations. The extent of the departure from the reciprocity law and the exact form of the function expressing the relation between E, I and t is still, however, subject to discussion.

Many workers in the field of photographic research have investigated this question and have arrived at different conclusions. The literature of the subject is too voluminous to permit of a complete analysis of methods and results at this time. It will be well, however, to review briefly the more notable of the researches along this line.

* Communication No. 193 from the Research Laboratory of the Eastman Kodak Company.

As stated previously, Scheiner (loc cit.) working on stellar photography obtained evidence which indicated that the reciprocity law was not valid and in 1893 Abney (loc. cit.) definitely verified this by sensitometric measurements. Abney concluded that the relation could be expressed in the form $E=It^p$ where p is a variable exponent. Schwarzschild (4) and Eder (5) confirmed Abney's conclusion that the simple relation $E=I\cdot t$ was not valid but from their experimental results concluded that the facts were represented by the equation $E=It^p$ where p is a constant. This constant they found to be approximately .86 for Schleussner gelatine emulsion plates.

A. Schellen (6) in 1898 concluded from his work that the reciprocity law was valid and that the facts were represented by the equation $E=I\cdot t$.

Englisch (7) in 1901 made measurements from which he concluded that Abney's result was correct, that is, that the exponent p must be a variable.

Sheppard and Mees (8) made some careful measurements and concluded that the failure of the reciprocity law is not a simple function of t and obtained measurements agreeing fairly well with those obtained by Englisch. They further concluded that the total value of $I\cdot t$ does not influence the extent of the failure and that the failure becomes noticeable at intensity values relatively proportional to the inertias of the plates.

Kron (9) in 1913 carried out a very extensive research on this subject and from his results obtained the following relation:

$$E=t\cdot I\cdot 10^{-a\sqrt{\left(\log\frac{I}{I_0}\right)^2+1}}$$

in which a and I_0 are constants applying to a given emulsion but are different for different emulsions. I_0 is termed by him the "optimal" intensity and is that intensity at which the blackening is a maximum for a given value of $I\cdot t$. Kron's measurements have been applied extensively by Halm (10) to the photographic photometry of stars and from his results it would appear that Kron's equation represents the facts satisfactorily.

More recently Helmick (11) has investigated this question and has found a very definite optimal intensity with a very marked failure for both high and low values of I in a relatively short range of I values. He has also determined the failure using monochromatic light and has presented equations showing the general relationship between E, I and t. Applications of these have been made to various problems of

photographic photometry. Ross (12) has discussed the theoretical aspect of this failure and points out the desirability of obtaining further reliable information in order to establish definitely the relationship existing between the various factors.

Work on this problem was started in this laboratory several years ago and a large amount of fairly reliable data was obtained. It was apparent from the values obtained at that time that the failure of the reciprocity law at least upon the particular materials being studied was much less than indicated by the results of previous observers. It was found that the equipment built for the work was not entirely satisfactory, the chief fault being a lack of sufficient range in the variation of I and t values. These data were not published and work was begun on a modification of the method and apparatus. A special sensitometer was designed and constructed which was described elsewhere (13). This instrument was completed some months ago and since that time work has been going forward and a large amount of data which is thought to be extremely reliable has now been obtained. Thus far, only three materials have been studied, these being Seed 30 plates, Seed 23 plates and motion picture (cine positive) film. The range of intensity and time over which observations have been made is somewhat greater than that used by previous observers. The work has been carried out with extreme care and it is felt that the data obtained are the most reliable thus far available on this subject. The various phases are discussed in detail in the following pages.

THE LIGHT SOURCE

The question of the most satisfactory source of radiation for use in exposing photographic materials was given very careful consideration. Incandescent electric lamps are by far the most satisfactory both in regard to quality of radiation and the precision with which they may be standardized and controlled. The lamps to be used were carefully seasoned and the voltage required for operation at a color temperature of 2500°K. was determined by comparison with temperature standards obtained from the Research Laboratory of the National Electric Lamp Association. According to Forsythe (14) color temperature measurements can be made to within ± 5°K. It has been found that the highest precision in color matching is obtainable with the Lummer Brodhum photometer head of the contrast type. This was used in color matching the various lamps against the color temperature standards, sets of readings being made by several different observers and values averaged. The lamps chosen were all of the

concentrated filament type, thus giving a source fairly small in linear dimensions and minimizing errors due to the failure of the inverse square law resulting from the finite dimensions of the source. Units of 100, 250, and 500 watts were used, the higher wattages being used for obtaining the high intensities and the 100-watt size for the low intensities.

In Table 1 are tabulated the data relative to the various lamps used. The candle power measurements were made on a 3-meter bench photometer using candle power standards obtained from the National Physical Laboratory. These standards operate at approximately 2500°K. and the color differences between these standards and the lamps used in this work were practically negligible thus giving the conditions for maximum photometric precision. Photometric readings were made by several observers in all cases and values averaged.

TABLE 1

Lamp No.	Watts	Volts	Effective Candle Power
B	250	102	72.8
A	250	110	89.4
2	100	81	1.14
4	100	85	1.03
5	100	81	1.39
6	100	79	1.40
12	500	81	131.2
7	100	79	1.40

While being measured each lamp was mounted in the lamp house in which it was later to be used. In the case of the 100-watt lamps which were used in a lamp house having a diffusing window which served as the effective source of light, photometric measurements in most cases were made on the lamp before placing it in the lamp house as well as after.

In Table 1 the values entered in the column marked "Effective Candle Power" are those obtained with the lamp in its housing. The greatest care was exercised in all photometric work and it is thought that the candle power values are correct to approximately $\pm 1\%$.

THE LAMP HOUSINGS

A detailed description of the sensitometer and its various parts will be found in another paper (15). It may be well, however, to state in

this place the reasons for using two lamp housings of essentially different types. The distance available in constructing the instrument was approximately 11 meters. Assuming that the light source should not be closer to the photographic material than 1 meter, a range of from 1 to 10 meters is available for varying the intensity by the use of the inverse square law. This corresponds to a range of only 1 to 100 in intensity. It was desired to vary the intensity over a range of approximately 1 to a million and for this a distance of 1000 meters would be required which is obviously prohibitive. It was therefore decided to use a lamp house carrying a diffusing window between the light source and the photographic plate. The area of this diffusing surface acting as a source can be varied to give a greater intensity range.

It is very important, however, that nothing be introduced which will cause a change in the quality of the light. All available diffusing material such as flashed and pot opal glass have selective transmission characteristics and are therefore useless for this purpose. A diffusing element was therefore made up using three sheets of white optical glass fine ground on both sides. These elements were separated by approximately 1 cm and the diffusion obtained was very satisfactory. In order to eliminate any errors due to the selectivity of this white optical glass, three exactly similar sheets were mounted in the other lamp house but without the surface grinding. The possibility of the presence of selective scatter in the case of the fine ground surface was considered and a test was made to determine if such an effect could be detected. Two lamps were carefully color matched against each other on the photometer bench. One of these was then placed in the lamp house with the diffusing window and the other in the lamp house with the clear window. The two were then placed in position on the photometer bar and adjusted for intensity balance. No lack of color balance was visible. Of course, this is not a definite proof that there is no selective scatter in the region of shorter wave lengths where the effect would have very little visual significance but a strong photographic effect. However, in making the exposures overlapping at the point in the intensity scale where the change from the clear to the diffusing lamp house was made gave excellent agreement photographically. The authors feel therefore that there is no selective scatter occurring with the ground glass window. At least the effect is so small as to be negligible both visually and photographically.

In the case of the lamp house having the diffusing window the effective area of the surface was limited by a set of diaphragms. The largest of these had an aperture of 100 sq. cm decreasing by one-half for each successive diaphragm. The effective candle power of this lamp house was carefully determined with the various diaphragms in position. Assuming precise cutting of the diaphragms, absolute uniformity of illumination over the diffusing surface, and complete diffusion within the effective limiting angle, candle power values should be directly proportional to the specified area. The photometric measurements which were made with extreme care show slight departures from the theoretical values. The actually measured values were used later in computing values of intensity. The relative candle power values for the diaphragms used are shown in Table 2.

TABLE 2

Lamp House No. 2

Diaphragm Number	Area	Relative Candle Power
	cm^2	
1	100.0	100.0
2	50.0	50.3
3	25.0	26.2
4	12.5	13.1
5	6.25	6.5
6	3.12	3.14
7	1.56	1.50
8	.781	.75

THE PHOTOGRAPHIC MATERIALS

There are a large number of photographic materials available differing in a marked degree in their sensitometric characteristics. It is probable that the extent of the failure of the reciprocity law is decidedly different for materials of different types. In order to obtain reliable results it is necessary in photographic research to average the results of a large number of observations. In this particular problem many of the exposures are very long and the accumulation of data is a somewhat slow and laborious process. To investigate a large number of materials is therefore a great undertaking and it was decided to limit the initial measurements to three typical materials and for this purpose the following were chosen: Seed 30 plates, representing a typical high speed material, Seed 23 plates, a medium speed material and cine positive film which has a relatively low speed.

All these are non-color-sensitive materials, that is, they are only sensitive to the shorter wave-lengths. For the time being therefore, the complication introduced by the sensitization for the longer wave-lengths is eliminated.

Different batches of the same material may vary to a certain extent from each other both in speed and in contrast. It is therefore necessary to make all measurements on a given material upon plates coated from the same lot of emulsion. A large quantity of each material was therefore ordered and reserved for this work. It is well known that the physical condition of the plate as regards temperature and humidity has a certain influence upon its sensitivity. In data obtained previously on this subject certain discrepancies were finally traced to differences in the humidity of the plate at the time of exposure. It is therefore considered necessary for sensitometric work of high precision to condition the materials before exposure. For this purpose a conditioning box was constructed. This is so designed that the humidity and temperature are precisely controlled by automatic means. After the adjustments have been made the conditions are maintained constant over long periods of time. The materials to be exposed in the reciprocity sensitometer were placed in this conditioning box and allowed to remain for 3 to 4 days before being exposed. It is well-known that even the most carefully manufactured plates show local variations in sensitivity from point to point on the surface (16). In order to increase the precision of the data, it is necessary therefore to multiply the number of determinations. Four sensitometric strips were exposed simultaneously. The density readings from these four strips were then averaged and a final curve plotted.

DEVELOPMENT

The density produced when a photographic plate is developed depends upon many physical and chemical factors. Among these may be mentioned the temperature of the developing solution, the time of development and the chemical constitution of the developing solution. There are other factors which influence the final density to a certain extent such as the kind of fixing bath used, the conditions of washing and drying the plates. These latter factors, however, have a relatively small effect and the conditions can be controlled so as to give practically no density variations due to these factors.

The elimination of variation in density arising from the time and temperature development and the constitution of the developer is

more difficult. All sensitometric strips were developed in a tray surrounded by a water jacket through which water at a constant temperature was continuously circulated. In this way variation in temperature was reduced to a minimum. The time of development was also in all cases sufficiently long so that it could be determined with considerable precision and variations due to this cause eliminated. Great care was used in making up the developing solutions in order to obtain an identical concentration of the various constituents. However, there seems to be a certain fluctuation in the developing power of the organic reducing agents and batches of developer made up at widely different times using developing agents from different lots show lack of constancy. In the work reported in this paper exposures on the same material were developed at widely different times and while every precaution was taken to obtain a constant degree of development, some variations occurred. In the opinion of the authors this is due rather to a fluctuation in the reducing power of the developing solution than to lack of control of the physical factors such as time and temperature.

DENSITY MEASUREMENTS

Density measurements were made with a high intensity densitometer described by one of the authors in a paper (17) on this subject. All density measurements were made with the photographic deposit in contact with a diffusing plate. The values obtained are, therefore, of diffuse density. As explained in the paper dealing with the instrument, the densitometer was calibrated by application of the inverse square law and it is felt that these readings are beyond criticism. Readings obtained with this instrument have been checked also against the Martens polarization photometer which has been in use for many years in reading densities. This latter instrument has proven entirely satisfactory for densities below 2.5 and within this region the two instruments agree to well within the photometric uncertainty.

EXPERIMENTAL PROCEDURE

The procedure for obtaining a group of strips exposed to a definite intensity (I) for a known time (t) was as follows: The lamp house was set in position and the distance between the source and the photographic plate adjusted. From the values of distance and effective candle power the intensity incident upon the plate was computed. After setting the lamp house, the diaphragms for the elimination of

scattered radiation within the housing were placed in position and the doors in the sensitometer housing closed to exclude the room light. The gear adjustment was made to give the desired angular rotation of the sectored wheel. The motor was then started and brought up to speed and checked against the standard clock. The plate-holder containing four strips of the material being studied was placed in position and the dark slide drawn. The operator by manipulation of the light control mechanism gave the exposure. The strips were then developed under standard conditions, washed and dried and the density values determined. As a rule, several sets of strips exposed at the various values of I were available for development at the same time. It is not feasible for an operator to handle more than about twelve strips at a single development. The group exposed together were therefore broken up and developed with strips exposed at other intensities. In this way variations due to extent of development were to a certain extent smoothed out. The density values obtained from the four strips exposed at a given intensity were averaged and the sensitometric curve plotted.

TABLE 3
Seed 30
Set No. 40

Strip No. 1	Strip No. 2	Strip No. 3	Strip No. 4	Average
1.63	1.60	1.58	1.66	1.61
1.31	1.37	1.31	1.44	1.36
1.10	1.19	1.11	1.16	1.14
.90	.99	.92	.93	.94
.68	.70	.68	.66	.68
.48	.48	.45	.43	.46
.24	.28	.29	.27	.27
.14	.17	.21	.17	.17

In Table 3 are shown the density values obtained from a group of four Seed 30 strips exposed under an illumination of 4.55 meter candles, the exposure time for the maximum step being one second. These values serve to illustrate the uniformity of density that can be expected in work of this kind. From the values obtained by averaging the four groups of readings, the curve shown in Figure 1 is plotted. This procedure was then repeated for various values of intensity and for each intensity used a curve similar to that shown in Figure 1 was obtained. For a given material, this group of curves

represents the basic data from which the relations existing between intensity, time, and exposure are later computed.

THE REDUCTION OF DATA

The elimination or minimization of systematic and accidental errors in photographic work is of great importance. It is common experience among workers in this field that the precise duplication of results is

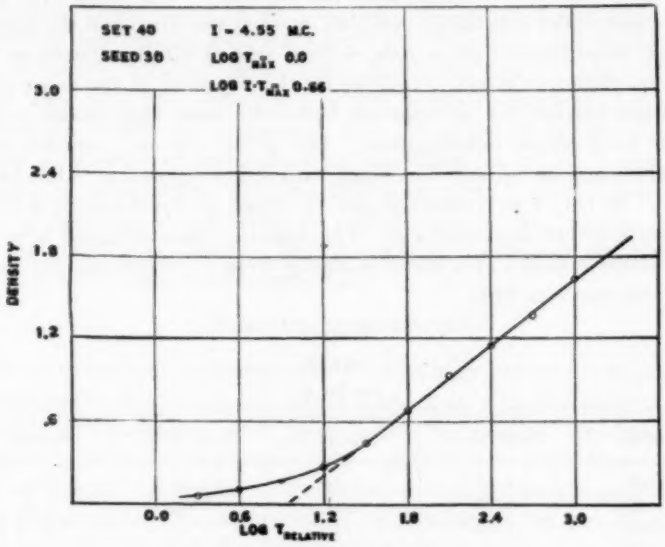

FIG. 1. *Typical H and D Curve.*

extremely difficult. These errors arise from many causes among which may be mentioned lack of absolute uniformity in the materials themselves and lack of complete control of various factors in the development, washing, fixing, and drying of the test strips.

Where high precision is desired, it is necessary to make a large number of observations from which the final and most probable result may be computed. The most satisfactory method of reducing such data is not always the determination of an arithmetical mean of a group of observed values. In some cases it is much better to smooth out variations by a graphic method. This is accomplished by plotting the observed values, drawing the most probable smooth curve as indicated by these points, and reading back from this smooth curve

the desired numerical values. The method of reducing the original data obtained from the test strips has been given very careful consideration by the authors, and it is felt that the method finally adopted reduces experimental errors to a minimum. Since the interpretation of photographic data is of prime importance, it is deemed advisable to present at this time a detailed account of the way in which the original data were treated. For this purpose all of the data and each step in its reduction to final form for one of the materials studied will be given.

TABLE 4
(Seed 23)

1	2	3	4	5	6	7	8	9	10
Set	Strips	Source	Lamp House	Aperture	Eff. cp	D(m)	I(mc)	T_{max}	$I \cdot t_{max}$ (mcs)
65	8	12	1	–	131.2	1	131.2	.25″	32.8
50	4	B	1	–	72.8	1	72.8	.50″	36.4
66	8	12	1	–	131.2	2	32.8	1.0″	32.8
51	4	B	1	–	72.8	2	18.2	2.0″	36.4
52	4	B	1	–	72.8	4	4.5	8.0″	36.4
96	4	7	2	1	1.4	1	1.4	32.0″	44.8
53	4	B	1	–	72.8	8	1.1	32.0″	35.3
54	4	4	2	1	1.03	1	1.03	32.0″	32.9
97	4	7	2	1	1.4	2	.35	2′8″	44.8
55	4	4	2	1	1.03	2	.26	2′8″	32.9
98	4	7	2	1	1.4	4	.088	8′32″	44.8
56	4	4	2	1	1.03	4	.065	8′32″	32.9
99	4	7	2	1	1.4	8	.022	34′8″	44.8
57	4	4	2	1	1.03	8	.016	34′8″	32.9
100	4	7	2	3	.367	8	.0058	2[illegible]16′32″	47.5
58	4	4	2	3	.270	8	.0040	2 16′32″	32.8
101	4	7	2	4	.183	8	.0029	4 [illegible]′4″	47.4
58A	4	4	2	4	.135	8	.0020	4[illegible]′4″	32.8
103	4	7	2	5	.091	8	.00069	18[illegible]12′16″	45.2

DATA RELATIVE TO SEED 23

In Table 4 is given a complete record of all exposure conditions for the tests made on Seed 23 plates. The numbers in the first column represent the arbitrary designation of the group of strips exposed under identical conditions, while in the second column is shown the number of strips comprising that group. In columns 3, 4, and 5 are the detailed specifications of the particular light source used, the lamp housing in which it was mounted, and in the case of the lamp house for the low intensities, the number of the diaphragm used to control the effective area of the source. In column 6, the numbers indicate the

effective candle power of the source assembly including the lamp itself, the lamp housing, and the limiting diaphragm. These values were determined directly on the bench photometer against reliable standards of candle power obtained from the Bureau of Standards and the Electrical Testing Laboratories. The utmost care was used in the photometric work and it is thought that all values presented are correct to within $\pm 1\%$. As stated previously, all lamps were carefully color matched against color temperature standards and it is estimated that the color temperature of the various sources is constant to within $\pm 5°$. In column 7, the numbers indicate the distance in meters between the light source and the plane occupied by the photographic plate during exposure. In the eighth column the values indicate the illumination in meter candles incident upon the exposure plane. In the ninth column, the values are the time of exposure for that step of the sensitometric strips receiving the maximum exposure. As explained in the paper (15), describing the sensitometer in detail, the sensitometric strip consists of ten possible steps, the exposure time varying by consecutive powers of 2 from step to step. For convenience in recording results, these steps are referred to by numbers from 1 to 10, 1 in all cases designating that step receiving the maximum exposure. In column 10 are tabulated the values of $I \cdot t$ in mcs for step No. 1 of the strip. It will be noted that the values in column 10 are only approximately constant. This exposure was found to give a strip consisting of nine measurable densities and amply sufficient for the establishment of the characteristic curve. It would have been necessary to double, approximately, the exposure in order to utilize the ten steps available and this was not considered necessary.

The density of the various areas of each strip was measured and the values for each group of strips were averaged. These values are tabulated in Table 5, the numbers at the top of each column indicating the corresponding step number on the strip. From these values a characteristic H and D curve was plotted for each group, the ordinates of these curves being density and the abscissae log exposure in m.c.s. It should be emphasized that the $\log E$ value (that is log of $I \cdot t$) for the various densities tabulated in column 1 of Table 5 is not constant. These density values when plotted at the proper $\log I$ position produce characteristic curves in such position that the variations in the values of $I \cdot t$ shown in column 10 of Table 4 are eliminated by reading from the plotted curves the density values corresponding to a constant value of $\log I \cdot t$.

From the characteristic curves plotted from the various groups of data shown in Table 5, the γ for each group was determined. This value of γ is an indication of the extent of development and should be constant for all groups if we assume that γ is independent of intensity. Gamma (γ) values for each group are tabulated in the last column of Table 5. An inspection of these shows that considerable variation occurs. As was pointed out previously, several weeks elapsed between

TABLE 5

(Seed 23)

Density (for the indicated step number on strips)

Set	1	2	3	4	5	6	7	8	9	γ
65	1.72	1.48	1.26	1.03	.79	.56	.33	.19	.10	.80
50	2.37	2.06	1.78	1.54	1.24	.96	.67	.40	.22	1.0
67	1.87	1.60	1.30	1.12	.88	.63	.38	.20	.11	.85
51	2.32	2.08	1.85	1.59	1.30	1.01	.72	.43	.24	.99
52	2.36	2.13	1.86	1.64	1.36	1.10	.76	.49	.28	1.03
96	2.06	1.90	1.69	1.50	1.26	.97	.70	.42	.23	.92
53	2.26	2.06	1.86	1.68	1.43	1.14	.86	.64	.42	.95
54	2.26	2.02	1.84	1.61	1.32	1.04	.74	.44	.22	1.00
97	2.00	1.82	1.70	1.48	1.24	.98	.71	.42	.22	.90
55	2.35	2.12	1.87	1.60	1.36	1.06	.75	.44	.21	1.05
98	1.96	1.84	1.66	1.44	1.20	.82	.68	.41	.21	.90
56	2.30	2.05	1.81	1.55	1.31	1.01	.70	.40	.20	1.04
99	1.94	1.68	1.47	1.20	.88	.56	.28	.14	.06	1.02
57	2.10	1.86	1.64	1.40	1.14	.86	.59	.34	.14	.95
100	2.10	1.99	1.76	1.52	1.25	1.00	.70	.40	.19	.97
58	2.14	1.93	1.70	1.42	1.11	.82	.52	.24	.10	.96
101	1.96	1.82	1.61	1.44	1.21	.96	.72	.46	.22	.90
58A	2.14	1.92	1.70	1.43	1.10	.80	.52	.29	.13	1.05
103	1.95	1.74	1.58	1.38	1.12	.85	.62	.35	.15	.89

Mean $\gamma = .95$

the beginning and conclusion of this work so that several different mixings of developer were used. The authors consider that a relatively large variation in γ is due to unforeseen variations in the developing solution. In Table 5, the results are arranged not in a chronological order in which the exposures were made, but according to a continuously decreasing value of I. The results in this table show no systematic variation of γ with intensity; however, it was considered desirable to make a specific test of this fact.

For this purpose, four strips were exposed at an intensity of 13.1 meter candles and another group of four strips at .0058 meter candles, the exposure time being adjusted so as to give approximately equal

exposure in the two cases. These eight strips were then placed in a tray and developed simultaneously, and the densities read and plotted. The results are shown in Figure 2, curve *A* being for the group exposed at high intensity and *B* for that exposed at a lower intensity.

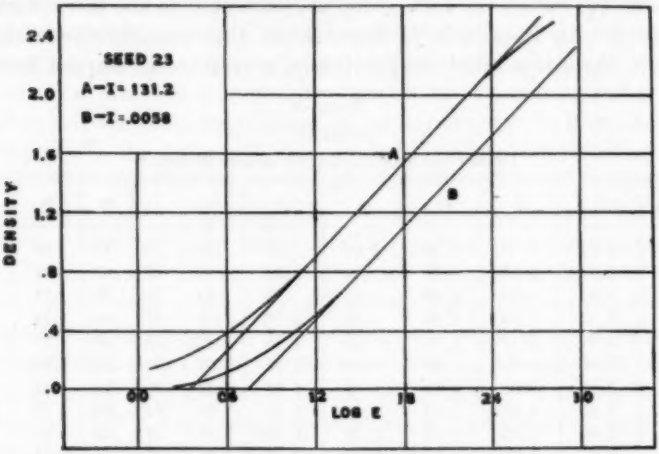

FIG. 2. *Characteristic curves for different intensities.*

It will be noted that there is no measurable difference in the slope of the two curves nor is there any indication that the curves differ in any way in general shape. This was repeated several times, in all some thirty or forty strips being exposed in groups at various intensities and developed together. For this particular material, it was impossible to detect any dependence of γ upon intensity. This fact being established it is necessary in some way to smooth out the γ variations occurring in Table 5. The mean value of γ for the entire group of determinations is .95. It is therefore necessary to correct each characteristic curve to that γ value.

Before passing on to the method for making these corrections, it may be of interest to consider briefly Figure 3. In this figure the observed γ values are plotted as a function of log intensity. The number adjacent to each point is that of the group to which the value applies. It will be noted that the γ values of groups 51 to 58 inclusive are practically equal. These numbers indicate the chronological order in which the exposure was made and also indicate roughly the order of development. These groups were probably all developed in the same mixing of developer and show no indication of any systematic

dependence of γ upon the intensity. Groups 65 and 66 show relatively low γ but as indicated by their number they were made at another time and probably show the effect of inconstancy in developer. Likewise groups Nos. 97 to 103 show relatively low γ, but they again probably represent different mixings of developer.

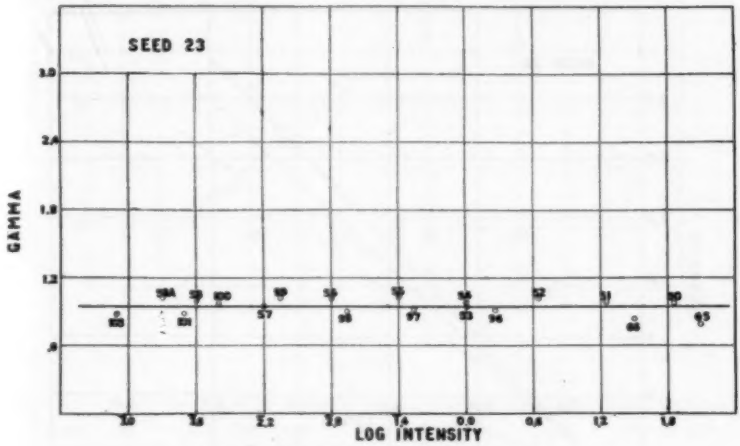

FIG. 3. *Variation of gamma with intensity.*

While the points, if considered entirely apart from other information, would indicate a slight decrease of γ for the very low and very high intensities, the authors consider that in view of other information and other tests there is no indication whatever of a dependency of γ upon intensity. The most probable γ-intensity curve is therefore a straight line parallel to the intensity axis and having a γ value of .95.

We will turn now to a consideration of the method for eliminating the effects of this accidental γ variation from the final results. This must be based upon a knowledge of the way in which the characteristic curve of the material changes with the extent of development. This subject has been studied in detail by many other observers and it has been shown by Nietz (8) that the straight line portions of the characteristic curves obtained for various kinds of development in a specified solution intersect at a point which may or may not lie on the log E axis. In order to determine the position of this intersection point for the particular developer and material under consideration, twelve strips were given identical exposure in the sensitometer. Three different times of development were chosen and four strips developed

for each of these times. The density values were read and averaged for each development time. The curves obtained are shown in Figure 4. This group of curves gives an intersection point at $D=-.30$. This result was checked by several other independent sets of observations.

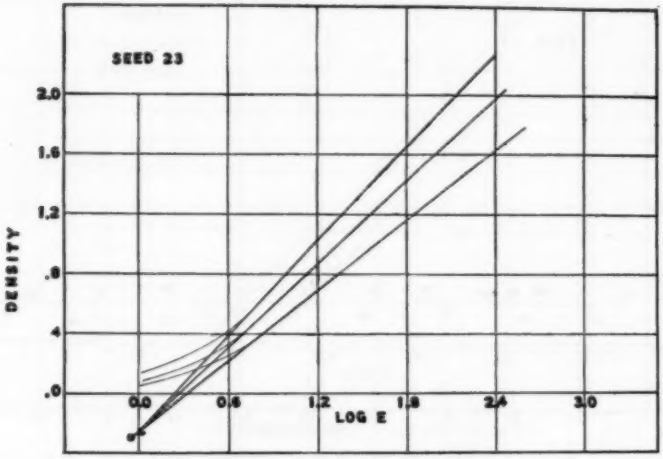

FIG. 4. *Position of intersection point.*

Having established the position of this intersection point for the material, we are now in position to correct the slope of each of the characteristic curves plotted from the data tabulated in Table 5. Referring to Figure 1, if the straight line portion be extended below the axis and on this extension a point be located at $D=-.30$ and through this point a straight line drawn having a slope of .95 (mean value of γ from Table 5), the line so drawn will be the straight line portion of the desired corrected characteristic curve. Using this straight line then as a basis, the corrected curve can be reconstructed. By proper averaging, it is possible to determine the mean shape of the characteristic curve as established by all of the data in Table 5. The shape and position of the toe and shoulder of the characteristic curve were determined in this way and drawn in proper relation to the straight line established as previously described. In Figure 5 the curve indicated by the dotted line (A) is that plotted from the original observations of set 56 and the solid line curve (B) is the corrected characteristic curve established for this group.

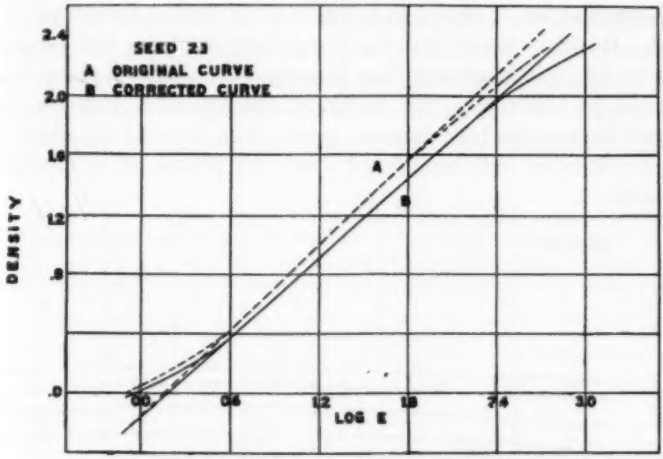

FIG. 5. *Method of correcting to mean gamma.*

TABLE 6
(Seed 23)
$I{\cdot}t$

Set	Log I	64	32	16	8	4	2	1	0.5	.25	.125
50	1.86	2.24	2.08	1.87	1.63	1.36	1.09	.80	.53	.27	.09
51	1.26	2.23	2.07	1.88	1.66	1.40	1.13	.85	.56	.30	.11
52	0.65	2.28	2.13	1.93	1.70	1.44	1.16	.88	.59	.32	.13
96	0.15	2.25	2.10	1.90	1.67	1.41	1.13	.85	.57	.30	.11
54	0.01	2.27	2.13	1.94	1.72	1.47	1.19	.92	.64	.36	.15
97	$\bar{1}$.54	2.25	2.10	1.91	1.67	1.41	1.13	.85	.57	.30	.11
55	$\bar{1}$.41	2.27	2.12	1.94	1.73	1.48	1.20	.92	.63	.36	.15
98	$\bar{2}$.94	2.23	2.07	1.87	1.64	1.38	1.11	.82	.52	.28	.10
56	$\bar{2}$.81	2.26	2.10	1.92	1.70	1.44	1.16	.87	.59	.32	.12
99	$\bar{2}$.34	2.26	2.12	1.92	1.70	1.45	1.17	.89	.60	.33	.13
57	$\bar{2}$.20	2.24	2.09	1.88	1.66	1.40	1.12	.84	.55	.29	.11
100	$\bar{3}$.76	2.20	2.03	1.83	1.59	1.33	1.04	.76	.48	.23	.07
58	$\bar{3}$.60	2.17	2.00	1.78	1.54	1.27	.99	.70	.42	.19	.05
101	$\bar{3}$.48	2.23	2.08	1.88	1.65	1.39	1.11	.83	.54	.28	.09
58A	$\bar{3}$.30	2.21	2.03	1.81	1.56	1.28	.99	.71	.42	.19	.05
103	$\bar{4}$.84	2.19	2.03	1.81	1.58	1.31	1.02	.74	.45	.21	.05

Mean $\gamma = .95$

The corrected characteristic curve for each group of strips was drawn and from them the density values tabulated in Table 6 were read. The numbers at the top of each column are the values of $I{\cdot}t$ to which the indicated densities apply. The step number on the strip has now been eliminated, and in this way the variations in the $I{\cdot}t$ value for a

given step number as shown in Column 10 of Table 5 have been eliminated. It will be noted that the first column in Table 6 is for an $I{\cdot}t$ value of 64; this is greater than any exposure actually given and is obtained by extending the corrected characteristic curve a short distance beyond the last observed point. The shape of the characteristic curve is so well established that this procedure is considered allowable.

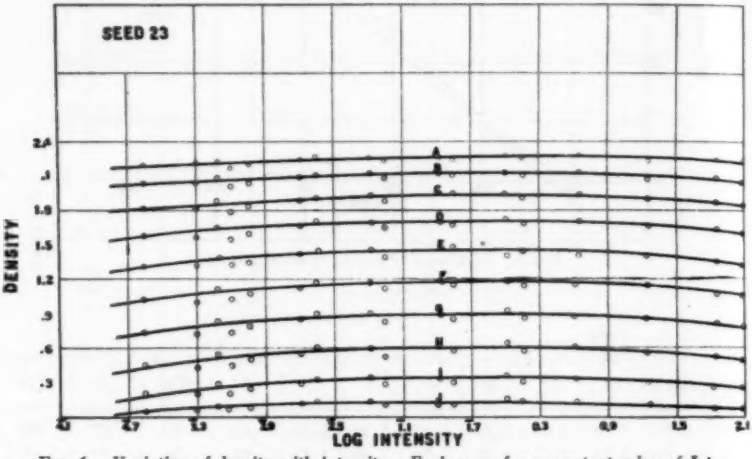

FIG. 6. *Variation of density with intensity. Each curve for a constant value of I·t.*

TABLE 7

Curve	$I{\cdot}t$
A	64
B	32
C	16
D	8
E	4
F	2
G	1
H	.5
I	.25
J	.125

From the data of Table 6, the curves shown in Figure 6 were plotted. These curves represent density as a function of log I for various constant values of $I{\cdot}t$. The values of the product $I{\cdot}t$ for the various curves indicated by the letters, A, B, C, etc. are shown in Table 7. In drawing these curves, a final smoothing out of the experimental

variations is obtained. In fact the actual points obtained from the data in Table 6 give curves with few irregularities. However, some slight variations were observable and in drawing the curves in Figure 6 the most probable position and shape was chosen. It is now possible to read back from the curves in Figure 6 the data tabulated in Table 8.

TABLE 8
(Seed 23)
Log t

Log I	I	$\bar{3}.0$	$\bar{3}.3$	$\bar{3}.6$	$\bar{3}.9$	$\bar{2}.2$	$\bar{2}.5$	$\bar{2}.8$	$\bar{1}.1$	$\bar{1}.4$	$\bar{1}.7$	Log t (sec.)
mc	mc	2^{-10}	2^{-9}	2^{-8}	2^{-7}	2^{-6}	2^{-5}	2^{-4}	2^{-3}	2^{-2}	2^{-1}	t (sec.)
2.1	2^7	.07	.24	.49	.79	1.06	1.34	1.60	1.86	2.04	2.22	
1.8	2^6	2.24	.08	.28	.53	.81	1.10	1.36	1.63	1.87	2.07	
1.5	2^5	2.09	2.25	.09	.30	.55	.83	1.12	1.39	1.65	1.89	
1.2	2^4	1.88	2.10	2.26	.10	.31	.56	.86	1.14	1.42	1.67	
0.9	2^3	1.68	1.91	2.11	2.26	.10	.32	.58	.87	1.16	1.43	
0.6	2^2	1.44	1.70	1.91	2.11	2.26	.10	.32	.58	.87	1.16	
0.3	2^1	1.17	1.44	1.70	1.92	2.12	2.26	.11	.33	.59	.88	
0.0	2^0	.88	1.18	1.45	1.70	1.92	2.12	2.26	.12	.34	.60	
$\bar{1}.7$	2^{-1}	.60	.88	1.17	1.45	1.70	1.92	2.12	2.26	.12	.34	
$\bar{1}.4$	2^{-2}	.34	.60	.88	1.17	1.44	1.70	1.92	2.11	2.25	.12	
		0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	Log t (sec.)
		2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	t (sec.)
$\bar{1}.1$	2^{-3}	.12	.33	.59	.88	1.16	1.44	1.70	1.92	2.11	2.25	
$\bar{2}.8$	2^{-4}	2.24	.12	.33	.59	.87	1.16	1.43	1.69	1.91	2.10	
$\bar{2}.5$	2^{-5}	2.10	2.24	.12	.32	.58	.86	1.15	1.43	1.68	1.90	
$\bar{2}.2$	2^{-6}	1.88	2.08	2.23	.11	.30	.56	.85	1.13	1.42	1.67	
$\bar{3}.9$	2^{-7}	1.65	1.87	2.07	2.22	.10	.28	.54	.83	1.12	1.40	
$\bar{3}.6$	2^{-8}	1.38	1.64	1.85	2.06	2.21	.09	.28	.51	.80	1.09	
$\bar{3}.3$	2^{-9}	1.06	1.35	1.62	1.83	2.04	2.20	.08	.23	.48	.78	
$\bar{3}.0$	2^{-10}	.74	1.03	1.32	1.59	1.81	2.02	2.18	.06	.19	.44	
$\bar{4}.7$	2^{-11}	.40	.70	.99	1.28	1.56	1.80	2.01	2.17	.03	.15	
		3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	Log t (sec.)
		2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}	2^{18}	2^{19}	t (sec.)

Certain definite values of I were chosen spaced at uniform intervals along the log I axis, the highest being log $I=2.1$ ($I=128$ mc). The interval used was log $I=0.3$ thus giving intensity values decreasing by consecutive powers of 2. These log I and I values are entered in the first two columns of Table 8. At each log I value, density values were read for each of the curves, A, B, C, etc. (Figure 6). The densities are tabulated in line horizontally with the corresponding log I and I values. The values of log t and t applying to each of the

densities thus obtained are indicated by the figures in the columns thus designated. The values of log t and t at the top of the table apply only to those densities down to and including the underscored value in each column. Values of log t and t in the middle of the table apply to those densities between the underscoring lines in each column; and those at the bottom to the densities below the underscored value in each column. From these data now, it is possible by reading along a horizontal line to obtain the characteristic curve determined with I constant and t variable, while by reading the perpendicular columns the characteristic curve for t constant and I variable may be plotted. The data contained in Table 8 and the curves in Figure 6 represent the final reduction of the observations.

In case there were no failure of the reciprocity law, the curves in Figure 6 would be horizontal lines parallel to the log intensity axis. It should be noted that there is a perceptible curvature to these lines, all being somewhat higher in the region of log $I = \bar{1}.7$ than at the higher and lower intensities. The departure from straight line form is not, however, very great considering the enormous range of intensity included. There also seems to be a distinct tendency for the curves in the intermediate density region to show a more marked departure from the straight line condition than those in the low and high density regions. While there is undoubtedly an intensity which gives a maximum density for constant $I \cdot t$, the location of the position of this so-called "optimal" intensity must in the case of this material at least be subject to great uncertainty.

DATA RELATIVE TO SEED 30

The method of reducing the data to final form for this material is very similar to that used with the data on the Seed 23 plates. In Table No. 9 are given the original density values. Each group consisted of from 4 to 8 sensitometric strips and the values shown are the averages for each group. The illumination incident on the plate during exposure is given in the column marked log I. The mean gamma values for each group are shown in the column thus designated, while in the last column marked $I \cdot t$ (max.) are the exposure values (in mcs) for step No. 1 of each group. It will be noted that there is some variation in the value of gamma. There is no doubt in the author's mind that this variation is attributable to the developing solution. It will be noted that the groups designated by consecutive numbers show very little variation in the gamma values.

TABLE 9
(Seed 30)
Density

Set	Log I mc	1	2	3	4	5	6	7	8	9	10	γ	$I \cdot t$ (max) mcs
61	2.11	1.61	1.43	1.28	1.10	.89	.68	.47	.34	.22	.13	.72	16.4
62	1.52	1.65	1.50	1.32	1.17	.97	.76	.54	.33	.22	.13	.72	16.4
39	1.22	1.54	1.37	1.17	.91	.65	.45	.27	.18	.12	.09	.82	4.1
63	0.91	1.34	1.20	1.02	.83	.57	.36	.20	.13	.08	.07	.74	4.1
40	0.63	1.61	1.36	1.14	.94	.68	.46	.27	.17	.11	.08	.79	4.1
64	0.61	1.35	1.16	.99	.81	.57	.36	.21	.12	.09	.08	.73	4.1
41	0.04	1.63	1.43	1.22	1.01	.76	.52	.28	.21	.12	.09	.84	4.1
41A	0.01	1.62	1.41	1.19	.97	.73	.47	.29	.18	.11	.08	.81	4.1
42	0.01	1.83	1.58	1.42	1.18	.97	.75	.48	.30	.17	.13	.79	8.2
89	$\bar{1}$.54	2.25	1.92	1.65	1.47	1.24	.97	.68	.37	.21	.14	.90	11.2
43	$\bar{1}$.41	1.69	1.51	1.34	1.16	.97	.75	.51	.33	.22	.13	.72	8.2
44	$\bar{2}$.81	1.70	1.54	1.26	1.09	.95	.68	.45	.28	.22	.13	.76	8.2
92	$\bar{3}$.76	1.98	1.72	1.60	1.37	1.18	.88	.56	.26	.15	.10	.97	11.9
93	$\bar{3}$.15	2.11	1.83	1.63	1.44	1.18	.84	.52	.29	.17	.12	1.03	11.5
94	$\bar{4}$.84	2.01	1.76	1.58	1.34	1.06	.75	.44	.23	.12	.10	1.00	11.3
95	$\bar{4}$.23	1.88	1.64	1.39	1.14	.84	.51	.26	.17	.10	.09	.96	11.1

Specific tests were made to determine whether or not any gamma variation due to the intensity factor could be measured. In Figure 7 are shown two characteristic curves, A, made with an illumination of 4.3 mc and B, with an illumination of .0014 mc. Several strips were

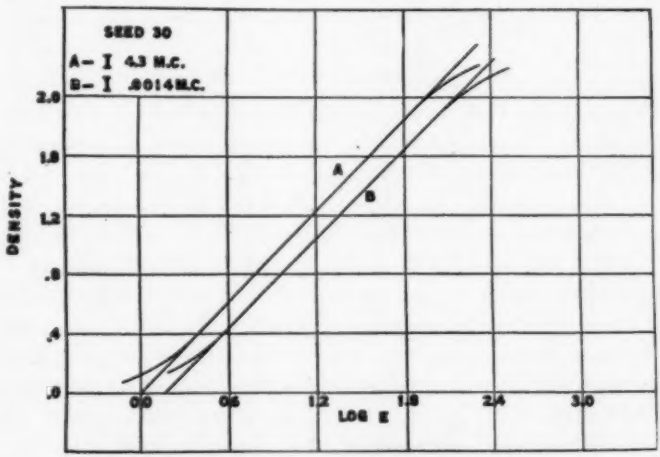

FIG. 7. *Characteristic curves for different intensities.*

exposed at each intensity and all strips of both groups developed simultaneously, the densities when read and averaged gave the curves shown. It will be noted that there is no detectable difference in the slope of the two curves. Several other direct tests of this point were made and in no case could any definite evidence of the dependence of gamma upon intensity be found.

It has therefore been considered allowable to average the values of gamma shown in Table 9, and to correct all of the various group curves to this gamma in order to smooth out density variations due to development differences.

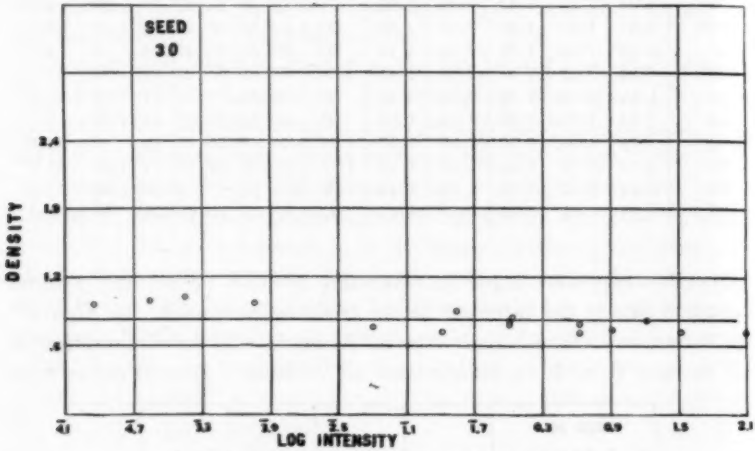

FIG. 8. *Variation of gamma with intensity.*

In Figure 8 are shown the gamma values plotted as a function of log intensity. The straight horizontal line is drawn at gamma equal to .83, the mean value from Table No. 9.

By a method exactly similar to that used in the case of Seed 23 material, the intersection point for the Seed 30 plate and the developing solution used was determined and from a mean of five groups of strips was found to lie at a value of $D=-.25$. By extending the straight line portion of the characteristic curve below the axis of zero density and locating on that line the point where $D=-.25$ and drawing through the point thus established a straight line having a slope of .83, the straight line portion of the corrected characteristic curve was established. By averaging values from the original curves the mean shape of the toe and shoulder was determined and by applying

these mean shapes to the established straight line portion the final corrected density-log E curve for each group was drawn. An example of this correction is shown in Figure 9.

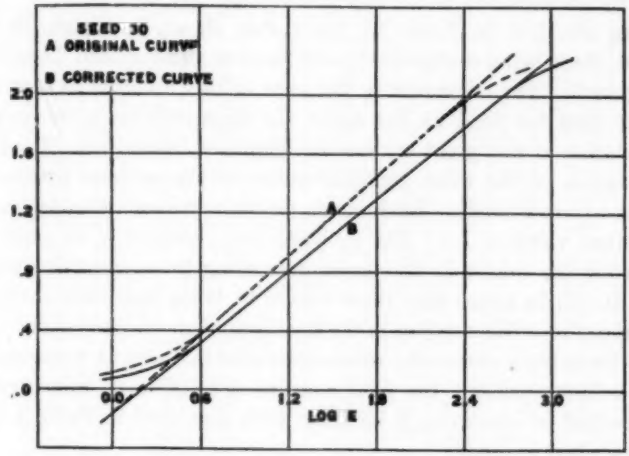

FIG. 9. *Method of correcting to mean gamma.*

TABLE 10
(Seed 30)
I·4

Set	Log I	16	8	4	2	1	.5	.25	.125	.0625
61	2.11	2.03	1.83	1.61	1.39	1.14	.89	.64	.38	.18
62	1.52	1.96	1.77	1.56	1.34	1.09	.84	.58	.33	.14
39	1.22	2.03	1.83	1.62	1.39	1.15	.89	.64	.38	.18
63	0.91	2.04	1.84	1.63	1.40	1.14	.89	.63	.37	.17
40	0.63	2.07	1.88	1.67	1.45	1.21	.94	.68	.42	.20
64	0.61	2.02	1.82	1.62	1.40	1.15	.89	.64	.38	.17
41	0.04	2.05	1.85	1.65	1.44	1.19	.93	.68	.43	.20
41A	0.01	2.03	1.84	1.64	1.43	1.18	.92	.68	.34	.21
42	0.01	2.03	1.83	1.63	1.40	1.06	.90	.65	.39	.19
89	$\bar{1}$.54	2.02	1.80	1.59	1.34	1.09	.83	.58	.33	.13
43	$\bar{1}$.41	2.09	1.89	1.70	1.49	1.25	1.00	.75	.50	.26
44	$\bar{2}$.81	2.05	1.86	1.66	1.45	1.21	.96	.71	.45	.23
92	$\bar{3}$.76	1.88	1.67	1.45	1.21	.96	.71	.44	.21	.08
93	$\bar{3}$.15	1.79	1.58	1.34	1.10	.84	.59	.32	.14	.04
94	$\bar{4}$.84	1.78	1.59	1.38	1.12	.87	.62	.36	.16	.06
95	$\bar{4}$.23	1.66	1.44	1.20	.95	.70	.44	.20	.06	.01

Mean $\gamma = .86$

From these corrected curves the density values tabulated in Table 10 were read. Points on the log E axis were so chosen that for all the groups density values apply to certain constant values of $I \cdot t$ as indicated by the numbers at the top of the respective columns.

From the data in Table 10, the curves shown in Figure 10 were plotted, these being similar in type to those shown for Seed 23 material in Figure 7. The curvature in the case of this material is somewhat greater than for Seed 23 but again the departure from the straight line relation is not great except for very low intensities. Here also the location of the most probable value of the optimal intensity is subject to uncertainty. Each curve shown represents the density for a constant value of $I \cdot t$. The $I \cdot t$ value for Curve A is 64 (mcs) the values for the others in the series decreasing by consecutive powers of 2. It will be noted that these curves fit fairly well with the points determined and this represents the final graphical smoothing out of the data. From these curves the values tabulated in Table 11 were read and these values represent the final conclusion relative to this material. The method of tabulating is identical with that used in Table 8 which was described in detail.

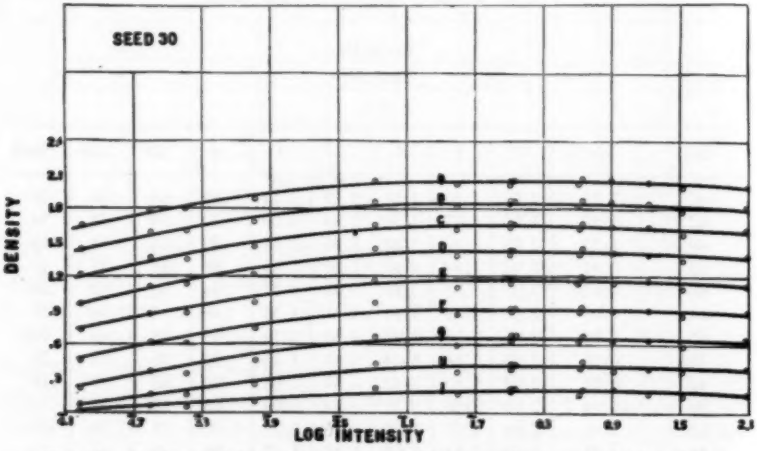

FIG. 10. *Variation of density with intensity. Each curve for a constant value of I·t.*

DATA RELATIVE TO MOTION PICTURE (CINE POSITIVE) FILM

The density values read from the sensitometric strips of this material are given in Table 12 with the values of illumination, gamma, and

TABLE 11
(Seed 30)

Log I	I	$\bar{4}.4$	$\bar{4}.7$	$\bar{3}.0$	$\bar{3}.3$	$\bar{3}.6$	$\bar{3}.9$	$\bar{2}.2$	$\bar{2}.5$	$\bar{2}.8$	Log t (sec.)
mc	mc	2^{-12}	2^{-11}	2^{-10}	2^{-9}	2^{-8}	2^{-7}	2^{-6}	2^{-5}	2^{-4}	t (sec.)
2.1	2^{7}	.16	.38	.62	.87	1.12	1.36	1.60	1.80	2.00	
1.8	2^{6}	2.01	.18	.39	.63	.88	1.14	1.38	1.61	1.81	
1.5	2^{5}	1.82	2.02	.18	.40	.64	.89	1.15	1.39	1.62	
1.2	2^{4}	1.64	1.83	2.03	.18	.40	.64	.90	1.16	1.40	
0.9	2^{3}	1.41	1.64	1.84	2.04	.20	.40	.65	.90	1.16	
0.6	2^{2}	1.16	1.41	1.64	1.84	2.04	.21	.41	.65	.90	
0.3	2^{1}	.90	1.17	1.42	1.65	1.84	2.04	.20	.41	.65	
0.0	2^{0}	.65	.90	1.16	1.42	1.65	1.84	2.04	.19	.41	
1.7	2^{-1}	.40	.65	.90	1.16	1.42	1.65	1.84	2.04	.18	
		$\bar{1}.1$	$\bar{1}.4$	$\bar{1}.7$	0.0	0.3	0.6	0.9	1.2	1.5	Log t (sec.)
		2^{-3}	2^{-2}	2^{-1}	2^{0}	2^{1}	2^{2}	2^{3}	2^{4}	2^{5}	t (sec.)
$\bar{1}.4$	2^{-2}	.18	.40	.64	.90	1.16	1.42	1.65	1.84	2.04	
$\bar{1}.1$	2^{-3}	2.03	.17	.39	.64	.89	1.15	1.41	1.64	1.84	
$\bar{2}.8$	2^{-4}	1.82	2.02	.16	.37	.62	.88	1.13	1.39	1.61	
$\bar{2}.5$	2^{-5}	1.59	1.80	2.00	.15	.34	.60	.85	1.11	1.36	
$\bar{2}.2$	2^{-6}	1.33	1.56	1.77	1.97	.14	.32	.57	.82	1.08	
$\bar{3}.9$	2^{-7}	1.04	1.29	1.52	1.74	1.94	.12	.28	.53	.79	
$\bar{3}.6$	2^{-8}	.76	1.00	1.24	1.48	1.70	1.90	.10	.25	.49	
$\bar{3}.3$	2^{-9}	.44	.70	.94	1.19	1.42	1.64	1.85	.08	.21	
$\bar{3}.0$	2^{-10}	.17	.38	.64	.89	1.13	1.36	1.59	1.80	.07	
		1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	Log t (sec.)
		2^{8}	2^{7}	2^{8}	2^{9}	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	t (sec.)
$\bar{4}.7$	2^{-11}	.05	.13	.33	.58	.83	1.06	1.30	1.53	1.73	
$\bar{4}.4$	2^{-12}	1.66	.03	.09	.26	.51	.77	.99	1.23	1.46	

$I \cdot t$ (max.) in the columns as indicated. An inspection of the gamma values show that they are fairly constant with the exception of the value for Set No. 84 which was exposed at the lowest intensity. Careful tests were made to determine whether or not this decrease in gamma was due to the low intensity or to other causes. In Table 13 are given the results of some of these tests. Set No. 1 consisted of two groups of strips, one exposed at an intensity of 131 mc and the other at .35 mc. These two groups were developed simultaneously, read and averaged. The gamma values obtained were 1.05 and 1.04, thus being equal to well within the limits of experimental error. However, when the intensity was reduced to .022 mc as in sets Nos. 2, 3, and 4, a lower value of gamma was in every case obtained for the low intensity.

Reducing the gamma value for the higher intensity to unity and expressing the mean gamma value at the low intensity as a decimal of that value, a mean of .86 was obtained. This is based on some 48 carefully developed sensitometric strips and considered very reliable.

TABLE 12
(Cine Positive)
Density

Set	Log I	Step Number									$I \cdot t$ (max)
	mc	1	2	3	4	5	6	7	8	γ	mcs
67	2.11	1.99	1.72	1.53	1.23	.94	.61	.37	.18	1.04	131.2
73	2.11	1.92	1.74	1.52	1.26	.98	.67	.39	.20	1.03	131.2
70	1.95	1.71	1.51	1.29	.99	.69	.44	.23	.10	1.01	89.4
76	1.95	1.82	1.61	1.36	1.09	.76	.47	.26	.12	1.00	89.4
68	1.52	1.57	1.24	.95	.65	.35	.17	.07	.04	1.03	32.8
74	1.52	1.71	1.41	1.20	.94	.65	.39	.21	.08	.94	65.6
69	0.91	1.29	1.01	.70	.42	.21	.09	.04	.04	.99	16.4
75	0.91	1.48	1.28	.98	.70	.39	.19	.09	.04	1.01	32.8
72	0.75	1.41	1.08	.80	.53	.28	.12	.06	.04	1.00	44.8
78	0.75	1.61	1.39	1.11	.81	.51	.26	.11	.06	1.00	44.8
79	0.15	1.63	1.32	1.08	.81	.50	.26	.11	.06	.96	44.8
80	0.15	1.56	1.28	1.02	.76	.51	.24	.12	.06	.94	44.8
81	$\bar{1}$.54	1.57	1.21	.92	.66	.38	.20	.10	.05	1.01	44.8
82	$\bar{2}$.94	1.40	1.12	.79	.48	.28	.14	.07	.04	1.05	44.8
84	$\bar{3}$.76	.78	.57	.34	.23	.12	.06	.04	.04	.80	46.6

TABLE 13
(Cine Positive)
Relation between Gamma and Intensity

Set	I	Log I	γ	%
1	131.0	2.12	1.05	100
1	0.35	$\bar{1}$.54	1.04	100
2	131.0	2.12	.96	100
2	0.022	$\bar{2}$.34	.92	96
3	131.0	2.12	1.00	100
3	0.022	$\bar{2}$.34	.80	80
4	131.0	2.12	.98	100
4	0.022	$\bar{2}$.34	.80	82
Mean	131.0	2.12	—	100
Mean	0.022	$\bar{2}$.34	—	86

Plotting the gamma values as a function of log intensity, the curve shown in Figure 11 is obtained. This is a straight line parallel to the log intensity axis for the great part of its length but there appears

to be no doubt that there is a definite decrease in gamma for the lower intensities. In correcting the original curves plotted from the data shown in Table 12, it is necessary to take into consideration this

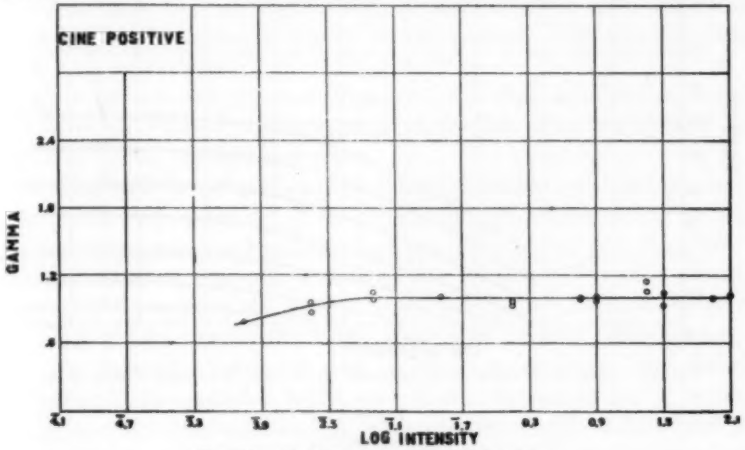

FIG. 11. *Variation of gamma with intensity.*

variation of gamma with intensity. The point of intersection of the straight line portions of curves for various times of development was determined as in previous cases and the corrected curves reconstructed by drawing straight lines through the points thus located. In the case

TABLE 14
(Cine Positive)

		$I \cdot t$							
Set	Log I	64	32	16	8	4	2	1	0.5
67	2.11	1.82	1.54	1.24	.93	.62	.32	.12	.01
73	2.11	1.89	1.60	1.30	1.00	.70	.40	.17	.03
70	1.95	1.79	1.49	1.19	.90	.59	.29	.11	.01
76	1.95	1.84	1.54	1.24	.94	.64	.32	.12	.01
68	1.52	1.72	1.42	1.12	.82	.53	.27	.09	.00
74	1.52	1.86	1.57	1.28	.98	.68	.38	.16	.03
69	0.91	1.85	1.56	1.27	.97	.68	.40	.16	.02
75	0.91	1.81	1.51	1.22	.92	.62	.36	.17	.02
72	0.75	1.85	1.56	1.26	.96	.66	.36	.15	.02
78	0.75	1.86	1.56	1.26	.97	.67	.37	.15	.02
79	0.15	1.88	1.58	1.30	1.00	.69	.38	.16	.03
80	0.15	1.90	1.60	1.31	1.01	.71	.41	.17	.03
81	$\bar{1}$.54	1.70	1.40	1.08	.78	.48	.18	.04	.00
82	$\bar{2}$.94	1.48	1.18	.91	.61	.32	.10	.00	.00
84	$\bar{3}$.76	.84	.61	.38	.16	.04	.00	.00	.00

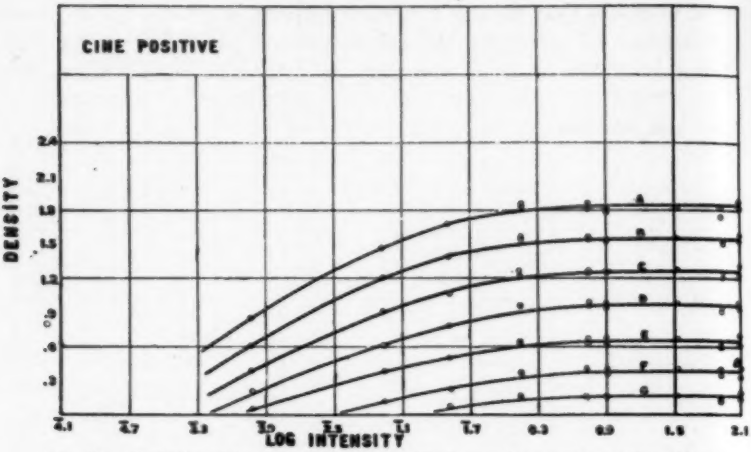

FIG. 12. *Variation of density with intensity. Each curve for a constant value of I·t.*

TABLE 15
(Cine Positive)

Log I	I	$\bar{3}.9$	$\bar{2}.2$	$\bar{2}.5$	$\bar{2}.8$	$\bar{1}.1$	$\bar{1}.4$	$\bar{1}.7$	Log t (sec.)
		2^{-7}	2^{-6}	2^{-5}	2^{-4}	2^{-3}	2^{-2}	2^{-1}	t (sec.)
2.1	2^{7}	.16	.39	.64	.96	1.27	1.55	1.85	
1.8	2^{6}	1.84	.16	.39	.65	.97	1.27	1.55	
1.5	2^{5}	1.55	1.85	.17	.39	.65	.97	1.27	
1.2	2^{4}	1.26	1.55	1.84	.17	.38	.65	.97	
0.9	2^{3}	.96	1.25	1.54	1.84	.16	.39	.65	
0.6	2^{2}	.63	.95	1.24	1.54	1.83	.16	.37	
0.3	2^{1}	.34	.62	.92	1.22	1.52	1.80	.14	
		0.0	0.3	0.6	0.9	1.2	1.5	1.8	Log t (sec.)
		2^{0}	2^{1}	2^{2}	2^{3}	2^{4}	2^{5}	2^{6}	t (sec.)
0.0	2^{0}	.12	.30	.59	.88	1.18	1.48	1.77	
$\bar{1}.7$	2^{-1}	1.72	.08	.26	.54	.82	1.13	1.44	
$\bar{1}.4$	2^{-2}	1.36	1.64	.03	.21	.48	.75	1.05	
$\bar{1}.1$	2^{-3}	.96	1.26	1.54	....	.15	.41	.66	
$\bar{2}.8$	2^{-4}	.57	.84	1.14	1.41	....	.08	.34	
$\bar{2}.5$	2^{-5}	.26	.47	.72	1.00	1.27	....	....	
$\bar{2}.2$	2^{-6}	....	.17	.35	.60	.85	1.31	....	
		2.1	2.4	2.7	3.0	3.3	3.6	3.9	Log t (sec.)
		2^{7}	2^{8}	2^{9}	2^{10}	2^{11}	2^{12}	2^{13}	t (sec.)
$\bar{3}.9$	2^{-7}	....	....	.08	.23	.44	.68	.92	
$\bar{3}.6$	2^{-8}	.74	....	....	....	.10	.29	.51	

of this material, however, the slope of this straight line is not constant but as indicated by the curve shown in Figure 11 for the various intensities. From the corrected curves thus drawn, the values tabulated in Table 14 were read for certain selected values of $I \cdot t$ and from these values the curves in Figure 12 were plotted. The curve marked A is for $I \cdot t = 128$ mcs.

From the smooth curves in Figure 12, the data tabulated in Table 15 were read. These values represent to the final reduction of the data relative to cine positive material.

The curves shown in Figure 6 (Seed 23), Figure 10 (Seed 20) and Figure 12 (cine positive) show graphically the extent of the failure of the reciprocity law for these three materials. From the data tabulated in Table 8, it is now possible to plot a series of H and D curves, one curve for each of the log I values shown in the column designated as log I at the extreme left hand of Table 8. Each of these curves is for the intensity indicated, the variation in exposure ($I \cdot t$) being obtained by the variation in the value of the time factor (t). A careful analysis of our results shows that when one curve is plotted for each of the log I values shown in Table 8 a family of curves is obtained in which the value of gamma is constant. This particular point has received careful consideration and the authors feel that there is no doubt in regard to the validity of this conclusion. This means that for this material at least all characteristic curves based on a time scale will have a constant value of gamma for designated conditions of development. If there is a failure in the reciprocity law, and the curves shown in Figure 6 prove conclusively that there is, it follows that a series of curves determined on the intensity scale basis must show a variation in the values of gamma, for fixed conditions of development. Since Schwarzschild's equation has been used so extensively, it seemed advisable to determine from our data the value of the exponent p appearing in the Schwarzschild expression, $E = It^p$.

As pointed out by Renwick[16] and Ross (loc. cit.) the difference in the gamma obtained by using a time and intensity scale may be used as a method of finding the value of the Schwarzschild constant p. The lateral shift of the characteristic curves (plotted for various values of I and with t as the variable) relative to the log E scale may also be used as a means for computing p. The latter method seems to be somewhat more direct and less subject to errors arising from the graphic determination of the gamma values. To determine the extent of the lateral shift, it was necessary to plot the characteristic curve

for each value of log I shown in Table 8, the density values necessary for this being obtained by reading the horizontal line of figures corresponding to the particular log I values. These curves were plotted and the straight line portion of each extended to cut the log E axis. The value of log E at the point where each curve intersects the log E axis was read and is tabulated in Table 16; in the column designated by the heading "log E_i." In sensitometric work it is customary to designate the value of exposure corresponding to these log E values, as the "inertia" and it is normally used as the number from which to determine the speed or sensitivity of the material. If there were no failure of the reciprocity law, the value of log E_i should be constant. It will be noted that this is not true. Since all of these curves have the same gamma but different values of log E_i, it follows that the reciprocity failure manifests itself, when the data are treated in this way, as a shift back and forth along the log E axis of a point at which the straight line extended cuts this axis. We may, therefore, use the variation in the value of log Ei as one method of specifying the extent of the failure of the reciprocity law.

It is well known that the slope of the straight line portion of the characteristic curve depends upon the extent to which development is carried. Hence the point of intersection of the straight line and the log E axis may be dependent upon development conditions. It has been shown definitely by Nietz (loc. cit.) that even in the presence of soluble bromides, the straight line portions of the characteristic curves obtained with different times of development intersects in a point which lies in general below the log E axis. The position of this point of intersection is one of the most characteristic factors of the photographic material and the particular developer used. A statement of the position of this intersection point may therefore be utilized as a means of specifying definitely the sensitivity of an emulsion, and the variation in the position of this point seems in this case to be the most satisfactory way of stating the failure of the reciprocity law.

For convenience, we will designate the point of intersection by stating the values of its co-ordinates, allowing b to represent the ordinate value of the point and log E_0 the abscissae value. It has previously been shown that for the Seed 23 plate and the developer used in this work $b = -.30$. Since the gamma is constant for all of the characteristics curves, it is only necessary to subtract a constant value from the values of log E_i shown in Column 2 of Table 16, in order to obtain the desired log E values for the intersection point.

The values are shown in the third column under the designation "log E_0."

TABLE 16
(Seed 23)
Gamma = .85 $b = -.30$

Log I	Log E_i	Log E_0	p	S
2.1	$\bar{1}.20$	$\bar{2}.88$	....	74
1.8	.16	.84	1.15	80
1.5	.13	.81	1.11	87
1.2	.10	.78	1.15	93
0.9	.09	.77	1.00	96
0.6	.09	.77	1.00	96
0.3	.08	.76	1.03	98
0.0	.07	.75	1.03	100
$\bar{1}.7$	.07	.75	1.00	100
$\bar{1}.4$	.08	.76	.97	98
$\bar{1}.1$	.08	.76	1.00	98
$\bar{2}.8$	.09	.77	.97	96
$\bar{2}.5$	.09	.77	1.00	96
$\bar{2}.2$	.11	.79	.94	91
$\bar{3}.9$	.13	.81	.94	87
$\bar{3}.6$	.16	.84	.91	80
$\bar{3}.3$	.19	.87	.91	76
$\bar{3}.0$	.22	.90	.91	71
$\bar{4}.7$	.28	.96	.88	61

From the values of log I and log E_0, it is now possible to compute for various intensity intervals the value of p, the exponent in the Schwarzschild equation. This law stated that in order to obtain a constant blackening $I \cdot t^p$ must be constant. This may be written

$$I_1 \cdot t_1^p = I_2 \cdot t_2^p$$

$$\log I_1 + p \cdot \log t_1 = \log I_2 + p \cdot \log t_2$$

$$p = \frac{\log I_1 - \log I_2}{\log t_2 - \log t_1}$$

By this equation p could be determined if the values of log t are known. These can be found but it is more convenient to use merely values of log E and log I as follows:

$$\log E = \log (I \cdot t)$$

$$\log E = \log I + \log t$$

$$\log t = \log E - \log I$$

$$\log t_2 - \log t_1 = \log E_2 - \log I_2 - \log E_1 + \log I_1$$

$$= (\log E_2 - \log E_1) + (\log I_1 - \log I_2)$$

$$p = \frac{(\log I_1 - \log I_2)}{(\log E_2 - \log E_1) + (\log I_1 - \log I_2)}$$

Using this equation, the values of p for consecutive intervals between values of log I were computed and are shown in the column designated as p, Table 16. It will be noted that these values range from 1.15 at the high intensities down to .88 for the lowest intensities. Our data, therefore, indicate very definitely that p is not a constant as was stated by Schwarzschild. In order to show in a different way the magnitude of the reciprocity failure, the values of sensitivity have been computed and tabulated in the column designated as S, Table 16. These values are obtained by taking the reciprocal of log E_0 and reducing these reciprocals to relative values, assuming a sensitivity of 100 at the position of optimal intensity.

TABLE 17
(Seed 30)
Gamma = .83
$b = -.25$

Log I	Log E_i	Log E_0	p	S
2.1	$\bar{2}.35$	$\bar{2}.05$	....	89
1.8	.33	.03	1.07	93
1.5	.32	.02	1.03	96
1.2	.32	.02	1.00	96
0.9	.31	.01	1.03	98
0.6	.31	.01	1.00	98
0.3	.30	.00	1.03	100
0.0	.30	.00	1.00	100
$\bar{1}.7$	.31	.01	.97	98
$\bar{1}.4$	.31	.01	1.00	98
$\bar{1}.1$	.32	.02	.97	96
$\bar{2}.8$	.34	.04	.94	91
$\bar{2}.5$	.37	.07	.91	85
$\bar{2}.2$	.41	.11	.88	78
$\bar{3}.9$	.46	.16	.86	69
$\bar{3}.6$	.51	.21	.86	62
$\bar{3}.3$	.57	.27	.83	54
$\bar{3}.0$	.64	.34	.81	44
$\bar{4}.7$	.72	.42	.79	38
$\bar{4}.4$	.80	.50	.63	32

This procedure for computing p and S was repeated with the data relative to Seed 30 and the values obtained are shown in Table 17. Here it will be noted that the value of p is also variable, ranging from .63 at the lowest intensity up to 107 at the highest. It is of interest to note that in the region from log $I = -3.0$ to -2.5 which are intensities such as to require exposure times ranging from a few minutes to several hours to obtain a density of unity, the mean value of p is approximately

.85. These intensities represent approximately those commonly met in stellar photometry and for that particular range the value of .85 for p agrees fairly well with that found by astronomical workers. For the same range of intensities for the Seed 23 plates, we find a somewhat higher mean value for p. The values of sensitivity (S) show very clearly that the failure of the reciprocity law for the Seed 30 material is considerably greater than in the case of Seed 23 plates.

TABLE 18
(Cine Positive)
Gamma variable with I
$b = -.34$

Log I	Log E_i	Log E_0	p	S	γ
2.1	$\bar{1}.96$	$\bar{1}.62$	....	100	1.00
1.8	$\bar{1}.96$	$\bar{1}.62$	1.00	100	1.00
1.5	$\bar{1}.96$	$\bar{1}.62$	1.00	100	1.00
1.2	$\bar{1}.96$	$\bar{1}.62$	1.00	100	1.00
0.9	$\bar{1}.96$	$\bar{1}.62$	1.00	100	1.00
0.6	$\bar{1}.97$	$\bar{1}.63$	.97	98	1.00
0.3	$\bar{1}.98$	$\bar{1}.64$	.97	95	1.00
0.0	0.02	$\bar{1}.68$	.88	87	1.00
$\bar{1}.7$	0.07	$\bar{1}.73$	.86	78	1.00
$\bar{1}.4$	0.13	$\bar{1}.79$	.83	68	.99
$\bar{1}.1$	0.21	$\bar{1}.86$	.81	57	.98
$\bar{2}.8$	0.31	$\bar{1}.95$	.77	47	.95
$\bar{2}.5$	0.42	0.04	.77	37	.93
$\bar{2}.2$	0.52	0.12	.79	32	.86
$\bar{3}.9$	0.65	0.24	.72	24	.80
$\bar{3}.6$	0.83	0.38	.68	17	.76

In Table 18, the values for the cine positive materials are tabulated. In this case the difference between log E_i and log E_0 is not constant. This follows from the variability of gamma found in the case of this material. From the values of gamma shown in the last column of the table and the value of b, it is possible to compute the difference between log E_i and log E_0 for each particular intensity, and in this way the values of log E_0 entered in the table were computed. From these values p was computed from the equation previously given. Here again we find the value of p to be variable. No values greater than unity were found but this is almost certainly because intensities sufficiently high were not available in making exposures. The failure of the reciprocity law in the case of this material is much greater than in the case of the other two. It will be noted that at the lowest intensity the sensitivity is only 17% of that obtained for the optimal intensity.

Had the values of log E_i (these being the values of log E where the density is equal to zero for each curve) been used in computing p, a different series of values for p would have been obtained. Likewise, had a series of log E values based on a comparison of exposures required to give a constant density (for instance, D=unity) been used, a still different series of values for p would have been obtained. Thus, if gamma is a function of intensity, p must depend upon the value of density chosen at which to evaluate this factor.

We have, therefore, in the case of cine positive a double variation in the value of p. All of our evidence, therefore, indicates that p can not be constant and that the Schwarzschild expression does not express adequately the relation between effective exposure, intensity, and time.

Thus far, the application of other analytical expressions has not been made to the data in this paper.

CONCLUSIONS

From this very careful study of the failure of the reciprocity law in the case of the three materials used, the following conclusions may be drawn. There is a definite failure of the reciprocity law. In the Schwarzschild expression, the exponent p is not a constant. The experimental determination of the value of optimal intensity must be subject to considerable uncertainty due to the extreme flatness of the curves. For Seed 23 and 30 plates, all characteristic curves determined on the basis of the time scale give equal gamma values when developed to the same extent, or, in other words, gamma is independent of intensity. In the case of cine positive material, it appears to be definitely established that gamma is to a certain extent dependent upon intensity.

The work done thus far leaves many questions relative to this subject unanswered and it is felt that a great deal more very careful experimental work is needed to solve the problem completely. The question of the dependence of the failure upon the wave length of the exposing radiation is of particular interest. Further work is at the present time in progress at this laboratory on this subject, and it is hoped in the future to present further data which will be of use in arriving at a satisfactory solution of this problem.

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ROCHESTER, N. Y.
SEPTEMBER, 1923.

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Seriengesetze der Linienspektren.—By F. Paschen and R. Götze. ii+154 pages. Springer, Berlin, 1922.

This book is indispensable to workers in atomic physics. It begins with an account of spectral series and series formulae, treated from a strictly empirical viewpoint. Practical methods for finding series and for calculating their limits are then discussed, and the nature of term multiplicities is described. The next section states the main results of the quantum theory of spectroscopy, with applications of inner quantum numbers to the explanation of doublet and triplet configurations. All this material is clearly written, but is so terse as to be contained in 21 pages.

The series tables follow. The numeration of spectral terms follows the Paschen system; this is praiseworthy because this system is so firmly intrenched in current usage. It would be well if future editions of Paschen-Götze and of Fowler's Report on series were to follow an example set in the latter author's paper on triply ionized silicon; both the principal quantum number and the usual term number were given in that paper.

In Paschen's tables the series run in horizontal rows. This is not so convenient to the eye as Fowler's arrangement in columns. Fowler

gives more complete discussions of the literature than Paschen-Götze, and has included the usual intensities and characters of spectral lines, together with many series formulae. Paschen and Götze give all the series of neon and all known series in argon, with a discussion of the inner quanta for neon, while Fowler devotes but little space to rare gases.

Paschen has noted many lines involving $\frac{R}{5^2}$ and $\frac{R}{6^2}$. The book gives a full account of the pp' and dd' groups found by Popow and Götze in alkaline earth spectra. Back's complete analysis of the beryllium spectrum is included, and some new classifications in Cd^+ are given. We shall not refer here to the isolated groups noted in the aluminium spark and in silicon, since complete classifications have recently been obtained by Paschen and by Fowler, respectively.

This book appeared just in time to miss the avalanche of relations discovered in spectra rich in lines. Future editions of books on spectral series will be inadequate unless they contain many energy-diagrams and "schemes" of multiplets; this is a case where printing expenditures should be lavish.

The series tables are followed by collections of term-magnitudes and differences of successive series terms. Tables of doublet and triplet differences would be very acceptable. A Rydberg interpolation table is given, based on the value $R = 109{,}737.1$; this is to be preferred to the value 109,678.3 adopted by Fowler. This latter table is not so conveniently arranged or so complete as the one in Fowler's report. The book ends with a table of the singlet, doublet, and triplet Zeeman types, including the approximate intensities of components.

The printing is clear, the binding strong, and the price moderate (less than three dollars gross).

It is only on looking through the references in books on spectral series that one can realize how much of the development of this field is due to Professor Paschen.

ARTHUR E. RUARK

COLOR MATCH AND SPECTRAL DISTRIBUTION

By W. E. Forsythe

When a black body is heated to a temperature of about 900°K it begins to give off very dark red light. As the temperature is raised the color of the emitted light changes first to a lighter red, then to orange, etc., approaching white more and more until at a temperature of about 5600°K its color matches sunlight as observed at the earth's surface. Practically all solids when sufficiently heated give off light that passes through the same series of colors. It has been found experimentally that the light from practically all radiating solids can be matched in color with the light emitted by a black body at some definite temperature. The color temperature of a particular source has been defined as the temperature of a black body which has the same distribution of energy in the visible spectrum as the source under consideration.

Such color matches can be easily and quite accurately made on an ordinary photometer bench using a Lummer-Brodhun contrast photometer having a contrast of about eight per cent. In a previous paper a very good agreement was shown[1] among the readings of a number of different observers in making color matches, and it was also shown that the color temperature scales used in the Nela Research Laboratory and the Bureau of Standards are in good agreement.

Color Matching by Red-Blue Ratio

If two sources so radiate that, when they are color matched by the method outlined above, they have the same spectral distribution to within the errors of observation, such color matches can be obtained by measurements of the relative brightnesses of the sources in two parts of the spectrum. If the two sources have the same relative spectral distribution when at a color match, the two curves that represent their energy distribution can be made to coincide exactly by changing the intensity scale of one of them. The measurements of the relative brightnesses can be made with any form of spectrophotometer or with an optical pyrometer using as the monochromatic screen first a red and then a green glass, or first a red and then a blue glass. To do this with the pyrometer it is more convenient to have the

[1] J. O. S. A. & R. S. I. *6*, p. 476; 1922.

pyrometer lamp calibrated for relative brightness, rather than for temperature.

To determine the relation between these brightness ratios and color temperature a series of red and blue brightness ratios for a black body at different temperatures was measured with a pyrometer calibrated on an arbitrary scale of brightness; then the logarithms of the ratios of the red brightness to the blue brightness (called the red-blue ratio) were plotted against the reciprocal of the temperatures. A very nearly linear relation was found. For a black body, if monochromatic radiation is used, a linear relation would be expected as can be seen from Wien's equation. Now if a particular red-blue ratio is obtained from measurements on a source being studied, its color temperature under this assumption of color match for the same red-blue ratio can be found from the point on this curve that has the same red-blue ratio.

In studying tungsten a similar series of red-blue ratios was measured and the logarithms of these ratios plotted against the reciprocal of the brightness temperatures and again the relation was found to be very close to linear.

The exact location of the curve through the experimental points was determined by means of a least square solution assuming a curve of the second order. This method was used because it was found rather difficult to decide just where the curve should be drawn. This same method was used to locate all the curves between the logarithms of the red-blue or red-green ratios and the reciprocals of the temperatures. By taking points on the curve for tungsten and the one for the black body that have the same red-blue ratio, the color temperature of the tungsten over a wide range was found under the assumption that the tungsten and black body are at a color match when they have the same red-blue ratio. Worthing using his determination[2] of emissive powers has calculated the relation between brightness temperature and color temperature. The results found in the present work were in very close agreement with the values found by Worthing. A weighted average of the two sets of results has been adopted as the best approximation to the correct relation.

A test of color matching by the red-blue ratio was made for a carbon lamp and a tungsten lamp several years ago by first color matching the two lamps by the use of the regular photometer and then determining when they were at a color match by the method of the red-blue ratios.

[2] Phys. Rev. *10*, p. 377; 1917.

When using the regular photometer the ends of the filament of each lamp were screened so as to eliminate errors due to end cooling. The results of this test are given in Table 1, in the first column of which are given the brightness temperatures of the carbon lamp for the different currents used. The results obtained by color matching the tungsten lamp with the carbon in terms of the brightness temperature of the tungsten lamp are given in column two. The results obtained for the brightness temperature of the tungsten for a color match with the

TABLE 1. *Brightness Temperature of Tungsten for Color Match with Carbon at Different Brightness Temperatures by Two Methods*

Carbon Brightness Temperature	Tungsten Color Matched on Photometer	Tungsten Color Matched Red-Blue Ratio
1736°K	1686°K	1685°K
1901	1735	1732
1862	1783	1780
1924	1821	1824
1982	1867	1862
2042	1915	1915
2097	1953	1955
2150	2006	2005
2202	2048	2042

carbon filament using the red-blue ratio are given in column three. These values are the averages of two sets of readings. The agreement is very good.

A tantalum lamp was color matched by direct comparison with the color standard using the Lummer-Brodhun photometer and also by the method of the red-blue ratios. The brightness temperatures obtained by the two methods for a number of color temperatures were in good agreement as can be seen from the results in Table 2. A test of the two methods for osmium also gave satisfactory results.

This method of the red-blue ratio was later applied to a number of different radiators to obtain their color temperature. Among the substances studied were both treated and untreated carbon, tungsten, tantalum, osmium, platinum, and the Nernst glower. The relation between the color temperature and the brightness temperature for the different radiators is shown in Table 3 where in column one are given the brightness temperatures and in the other columns the corresponding color temperatures for this brightness temperature of the different radiators.

COLOR MATCH AND SPECTRAL DISTRIBUTION

When two different radiators such as tungsten and a carbon lamp are color matched, i.e., adjusted so that the integral color is the same, there has been some question as to the extent to which they have

TABLE 2. *Brightness Temperature of Tantalum for Particular Color Temperature By Two Different Methods of Color Matching*

Color Temperature °K	Brightness Temperature	
	Color Matched on Photometer	Color Matched by Red-Blue Ratio
1819	1647	1648
1947	1748	1750
2061	1838	1841
2171	1924	1928
2275	2008	2007
2376	2086	2085
2475	2160	2158

TABLE 3. *Relation between Brightness Temperature and Color Temperature for a Number of Substances*

Brightness Temperature	Corresponding Color Temperature for						
	Untreated Carbon	Gem	Platinum	Nernst Glower	Osmium	Tantalum	Tungsten
1400°K	1414	——	1568°K	1538	1444	1507	1492
1500	1515	——	1692	1642	1562	1631	1607
1600	1616	1620	1821	1747	1680	1758	1723
1700	1718	1735	1952	1852	1799	1883	1841
1800	1820	1852	2086	1954	1919	2010	1961
1900	1923	1962	——	2053	2045	2137	2082
2000	2028	2064	——	2146	2168	2265	2206
2200	2240	2255	——	2310	2427	2500	2457
2400	——	——	——	——	2688	2785	2718
2600	——	——	——	——	——	——	2988
3000	——	——	——	——	——	——	3564

the same distribution of energy in the visible spectrum. This was tested out in this laboratory[3] some years ago for carbon, tungsten, and tantalum for a single color temperature in the neighborhood of 2160°K. A carbon lamp was taken as the standard and a tungsten and tantalum lamp color matched with the carbon lamp using the ordinary contrast

[3] Hyde. Astrophys. Jour. *36*, p 114; 1912.

photometer. The relative distribution of energy in the visible spectrum of the three lamps at the voltages found for color match was then measured using a spectrophotometer. Evidence was found that when the radiation intensities were set relatively equal near the two ends of the visible spectrum (i.e. $\lambda = 0.5\mu$ and $\lambda = 0.7\mu$) the tungsten was about one half per cent brighter at the center of the spectrum ($\lambda = 0.58\mu$) and the tantalum was about one per cent brighter at the center of the spectrum ($\lambda = 0.58\mu$) than it should be for perfect color match with the carbon.

A test of the spectral distribution for a color match for several different radiators was made over a wide range of temperature by measuring the brightness in three parts of the visible spectrum using a disappearing filament optical pyrometer, with a red, ($\lambda =$ ca. 0.66μ), a green ($\lambda =$ ca. 0.54μ), and a blue color ($\lambda =$ about 0.47μ) screen in turn as the monochromatic screen.

When using a colored screen having a rather wide transmission band it is often required to consider results thus obtained in connection with a definite wave-length. To do this it is necessary to consider the effective wave-length of the screen. The effective wave-length of a screen having a wide transmission band has been defined as the wave-length such that for any definite temperature interval for a black body the ratio of the radiation intensities for this wave-length shall equal the ratio of the integral luminosities through the screen used. Since the measurements compared were all made at the same spectral distribution, corrections were not necessary for the change in the effective wave-length of the different screens.

The sources studied were color matched by the method of the red-blue ratio described above and then the red-green ratios measured for the same temperature range. The logarithms of the red-green ratios (all ratios corrected, of course, for the transmission of the lamp bulb) were plotted against the reciprocal of the color temperatures as obtained from the red-blue ratios and again very nearly a linear relation found.

The red-blue ratio for the black body was measured from $T = 1336°$K to $T = 2800°$K. These readings were obtained by making observations on several platinum wound black bodies, a carbon tube black body and a black body which consisted of a hollow tungsten filament with small radial holes designed by Worthing of this Laboratory.[4] The

[4] *loc. cit.*

red-blue ratio for tungsten extended from a brightness temperature of about 1400°K to about 3000°K. Due to the difficulty in making low temperature readings in the blue part of the spectrum no weight was given to points below 1660°K in determining the equation of the curves.

If the different radiators have the same spectral distribution in the visible spectrum when they are color matched, the logarithms of the red-green ratios for the different substances should all lie, along the same straight line when they are plotted against the reciprocal of the color temperature as found from the red-blue ratio. The reason for this is that the color temperature could be obtained just as well from the red-green ratio as from the red-blue ratio and if color match means the same spectral distribution these color temperatures should be the same. On the other hand if the values of the logarithms of the red-green ratios do not all lie along the same curve their distance from the curve for the black body will show how the intensity in the green part of the spectrum for the different radiators varies from that of the black body for the same red-blue ratios. That this is the case can be seen when it is remembered that for any red-blue or red-green ratio the same value for the red brightness is used for each particular radiator and the same relative values for different radiators.

The data on the red-green ratio for carbon when plotted against the reciprocal of the red-blue color temperature fit the black body data within the experimental error. This means that when the carbon is color matched by means of the red-blue ratio it also fits the black body curve in the green. A color match, therefore, between carbon and a black body indicates the same spectral distribution. The values of the red-green ratio for tungsten do not fit the black body curve. The observations show that tungsten radiates in such a manner that when its temperature is set to give the same red-blue ratio as the black body for a particular temperature the green part of the tungsten spectrum is relatively brighter than that of the black body. This difference is, however, small being less than ½% at a color temperature of 1600°K and increasing to about 1% at a color temperature of 2600°K. For tantalum the difference is in the same direction, that is, the green part of the spectrum of tantalum is relatively brighter than that of the black body when the tantalum and black body are set at the same red-blue ratio. The relative difference is about constant and equal to about 1% difference in brightness. This agrees with

a previous test in this Laboratory.[5] The results for platinum are about the same as those for tungsten. Osmium, on the other hand, when its temperature is set to give the same red-blue ratio is relatively less bright in the green than the black body. This agrees with a former observation[6] that osmium radiation appears slightly purple while that from tungsten appears greenish.

The red, green, and blue brightness of the Welsbach gas mantle was also measured. The red-blue ratio thus found corresponded to a color temperature of about 2800°K. The measurements showed that for the mantle the green brightness was about 30% too high for an exact color match. That is, if the curves that represent the spectral distribution of a black body at 2800°K and the one that represents the spectral distribution of a Welsbach mantle be made equal in the red ($\lambda = .665\mu$) and in the blue ($\lambda = .467\mu$), the two curves would differ in the green where intensities for the Welsbach mantle are about 30% higher than those for the black body.

When it was found that the results of color matching by the red-blue ratios agree as nearly with values obtained by using the Lummer-Brodhun contrast photometer and also that the results on the red, green, and red-blue ratios showed but a small deviation from actual color match, the method of red-blue ratios was used to extend the color temperature scale. By the use of the tungsten tube the red-blue ratio for the black body was extended up to about 3000°K. Using gas-filled tungsten lamps it has been possible to extend the color temperature scale of tungsten up to about the melting point of tungsten, that is, to about 3600°K.

NELA RESEARCH LABORATORY,
NATIONAL LAMP WORKS,
CLEVELAND, OHIO.
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Loss and Gain of Electrons by Fast-Moving Hydrogen Atoms (Hydrogen Canal Rays).—According to Wien's conception (cf. "Notes" This Journal, *7*, p. 373) a beam of canal rays passing through a gas consists partly of neutral and partly of ionized atoms, the relative proportions of the two kinds tending toward a statistical equilibrium. The free paths traversed by neutral atoms before they are ionized by losing electrons are distributed exponentially about a mean value L_2, like the free paths of gas molecules between collisions; similarly the

[5] Hyde. Astrophysical Journal, *36*, p. 114; 1912.
[6] Hyde. J. Franklin Inst. *170*, p. 32; 1910.

free paths of ionized atoms before they pick up electrons and become neutralized are distributed about a mean value L_1. Rüchardt measures L_2 directly on a beam of hydrogen-atom canal rays of uniform speed, carefully separated out from a heterogeneous beam by electric and magnetic deflections. All the charged atoms are removed from the beam by sending it through a narrow intense electric field; it then travels across a region of width x to a thermal receiver, the reading of which gives the total number N of particles in the beam; an electric field is then applied and sweeps out the particles which are ionized in the region x; the number of particles thus lost from the beam is set equal to $N \cdot \exp(-x/L_2)$, and L_2 is calculated. Its reciprocal increases with the pressure of the gas according to a function $1/L_2 = a + b_p$. Rüchardt assumes that the term a is due to residual vapors from stopcock grease, etc., and shows that it vanishes in a specially made tube without greased joints. At higher values of p (.014 mm and greater) it is somewhat vaguely assumed that the vapors are negligible, and the radius of the effective cross section of the gas molecules is calculated from L_2 in the usual manner. The values are about .7 A in H_2 gas, about 1.1A in nitrogen and oxygen. They increase slightly as the speed of the canal rays is diminished. The ratio L_1/L_2 is equal to the ratio of ionized to neutral atoms in the undisturbed beam; this is determined from the readings of the thermal receiver with the aforesaid narrow intense field alternately off and on. From this L_1 is calculated; it is the same for all three gases, is distinctly smaller than L_2, depends on pressure in the same way, and increases rapidly with increasing speed of the canal rays.

The relation between L_1 and the speed v of the canal-rays is of the form $1/L_1 = av^{-4} - b$ where a and b are positive constants; suggesting that the condition for an electron being caught by a nucleus is that the nucleus must pass it at a distance r lying between a minimum limit r_1 independent of v and a maximum limit r_0 varying inversely as v^2. Rüchardt attempts a theory of this type in the Zeitschrift für Physik; but quantitative agreement cannot be obtained except by assuming two electrons in each gas molecule, whether H_2, N_2 or O_2.

Observations on a beam of hydrogen canal rays not sifted out in the indicated manner lead to qualitatively similar results. Discordant results of earlier investigators are ascribed partly to their use of heterogeneous beams and partly to stray magnetic fields. [E. Rüchardt (Munich) Ann. d. Phys. *71*, pp. 377–423; 1923. Zs. f. Physik, *15*, pp. 164–171; 1923.]

K. K. DARROW

STUDIES BASED ON THE SPECTRAL COMPLEMENTARIES

I. Subjective Saturation of Spectral Hues

II. Individual Variations of the Normal Color-Sense

By R. H. Sinden

Introduction

The lack of a group of individual data pertaining to the wave-lengths of spectral complementary hues which should be strictly comparable and should correspond to a white of definitely known spectral constituency has been deplored by recent writers in physiological optics. Accurate comparative data on the luminosity ratios of complementaries is wholly lacking. In view of the ever-increasing importance which is being attached to the precise specification and measurement of color, accurate data pertaining to fundamental characteristics of the color-sense, such as complementary-hue data, is, *a priori*, almost certain eventually to be of value. The opportunity for a comparative study of the normal color sense afforded by such data has proved of unexpected interest.

The general mode of procedure which has always been followed in investigations of the spectral complementaries, has been to provide a photometric field one side of which can be uniformly illuminated by two superposed pencils of more or less nearly homogeneous light of arbitrary hue, the other side by a white of definite continuous spectral character. There are then five variable conditions, namely, the representative wave-lengths and the intensities of the spectral components, and the intensity of the standard white; and two of them may be arbitrarily fixed at the start, the observer varying the other three until the two sides of the photometric field appear as near alike as possible. The finished observation may be completely described by recording three figures, to wit, the representative wave-lengths of the spectral rays, and the ratio of their intensities.

The present experimental investigation of the spectral complementaries differs from preceding investigations in the following particulars:

1) A group of individuals has been examined under identical experimental conditions.

2) In addition to complementary wave-lengths, the luminosity ratios of complementaries have been determined for some of the observers.

3) Important experimental details, such as size and absolute brightness of the photometric fields and character of the photometric junction, have been carefully regulated and specified.

4) The quality of the comparison white has been fully standardized.

The chief distinctive outcomes of this investigation have been 1) the derivation of numerical values for the relative subjective saturations of the spectral colors, based upon the luminosity ratios of complementaries, and 2) a study of the individual differences of the normal color sense.

APPARATUS

The apparatus used is shown diagrammatically in Fig. 1. *ABC* is a Brace spectrophotometer. The 60° prism, *P*, of this well-known instrument is made up of two 30° parts cemented together with Canada

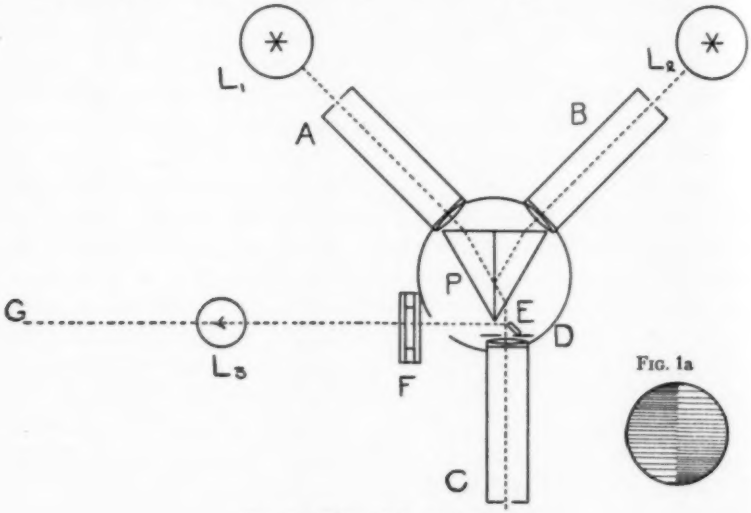

FIG. 1. *Color-mixing apparatus.*

balsam, one of the abutting faces normally carrying a narrow horizontal strip of silvering. For the present purpose the strip of silvering is replaced by a semi-transparent silver film covering the whole area of the abutting face. It will be readily seen that a mixture of the beams of light from the two collimators *A* and *B* is effected by the half-silvering. Perfect uniformity of the mixture over an extended area was gained by a chemically deposited film, after having freshly polished the glass

surface upon which it was laid. Visible interference fringes, due to a slight difference in the refractive indices of the Canada balsam used to cement the halves together, and the glass of the prism, were eliminated by placing a fine, straight wire between the lower edges of the abutting faces, thus giving to the thickness of the balsam layer a relatively steep gradient without causing a doubling of spectral lines seen through the prism, which occurred if the wire was placed vertically.

A white comparison field is provided by a plane surface, E, coated with magnesium oxide, inclined at 45° to the axis of the ocular, C, illuminated from a direction perpendicular to the axis of C. The inner vertical edge of E intersects the axis of C. This edge is sharp, and capable of complete disappearance under conditions of photometric identity. The technique by which it is produced is due to Dr. A. H. Pfund. A piece of plate glass is "smoked" with magnesium oxide, a crack is started in it by a short scratch near one edge with a glass-cutter, and the glass is then broken by steady pressure. The break will generally run straight across, leaving a perfectly sharp edge beyond the glass-cutter scratch.

For the illumination of E, a hole was cut in the side of the otherwise light-tight prism-box. The illumination is accomplished by means of a small gas-filled tungsten lamp, of the automobile headlight type, having a short coiled V-shaped filament (L_2, Fig. 1). The lamp is mounted on a photometer car which runs on an optical bench in the direction EG. Immediately outside the prism-box, in the path of the light from L_2 to E, is placed a filter F, for the purpose of modifying the spectral energy-distribution of the source L_2.

The filter is of a type recently developed by Dr. Pfund, and is capable of reproducing the spectral distribution of sunlight very nearly when used in connection with a suitable source. It consists of a liquid-cell having two distinct compartments, one of which contains an ammoniacal solution of copper sulphate, the other an aqueous solution of the sulphates of copper and cobalt. The proper concentrations of the different solutes give a smooth transmission curve which is almost the reciprocal or mirror-image of the energy distribution curve of the source.

The spectral distribution of average noon sunlight at Washington (Abbot[1]) was adopted as standard. The energy distribution of the artificial source as compared to that of the adopted standard is shown in Fig. 2. The former was obtained by taking the products of the transmission of the filter, measured spectrophotometrically, and the

energy distribution of the source L_3 determined from its color-temperature, a "Planckian distribution" being assumed.

The color temperature of L_3 was maintained at approximately 3000°K. The constancy of the color-temperature of the source during the series of experiments was checked by means of another lamp of the same type which was preserved as a standard. The transmission of the filter was also checked and was found not to have changed at the completion of the experiments, three months after it was first sealed.

An actual comparison of the hue of the artificial source with that of noon sunlight at Baltimore in April, showed no appreciable color-difference.*

Relative intensities of illumination on E were determined by the inverse square law, which was checked for this particular set-up and found to hold certainly within the required limits of accuracy. A rough estimate of the absolute intensity has been obtained from the rated candle-power of L_3.

The photometric field is restricted by a diaphragm with circular aperture, D, Fig. 1, placed immediately in front of the ocular lens, for which purpose three interchangeable diaphragms are provided, having apertures subtending respectively at the ocular slit angles 1°, 1½° and 2°. The 1½° field was used in all the final observations. The ocular slit is fixed in dimensions, being 3.2 mm by 0.43 mm.

The spectral sources L_1 and L_2 are 50-watt gas-filled tungsten lamps with milk-glass bulbs, bearing the trade-name "Colonial Mazda." These lamps, together with L_3, were operated on the same 120 volt storage-battery circuit, so that relative variations in intensity were negligible.

The intensities of the spectral stimuli are adjusted by varying the widths of the collimator slits. Wave length adjustments are accomplished by rotation about the axis of the prism of collimator B and the ocular C, both being provided with tangent screws. Wave length determinations are made by means of an angular scale with a vernier associated with C. The smallest division of the vernier is 1/100 degree, amounting to something less than 1mμ on an average throughout the spectrum. Since B is moveable and has no scale, it is necessary to provide in the spectrum of B a fixed line to serve as a

*N. B. Data on the color temperatures of sunlight recently obtained by Priest[3] show that the color temperature of average noon sunlight at Washington is approximately 5000°K, thus this value may be taken as approximately specifying the standard white here used.

fiducial mark for wave-length measurements: conveniently accomplished by introducing before the slit of *B* an absorption cell containing a solution of neodymium chloride. An accessory eyepiece is used for setting on the absorption band, and for calibrating the instrument from known line spectra.

The ideal type of spectral stimuli would be those whose entire range of wavelengths lie within the threshold of hue difference discrimination. Such stimuli, although not necessarily objectively homogeneous, could be appropriately called "monochromatic." With the present apparatus, however, there were practical difficulties in the way of realizing monochromatic stimuli, and it seemed simpler to correct for possible errors introduced by the use of wider spectral bands. Errors of two types might thus arise:

(1) The dominant hue of the stimulus composed of the wide band may not coincide with that of the monochromatic stimulus having the same mean wavelength.

(2) It is possible that the subjective saturation of the composite stimulus may suffer a diminution, thus affecting the complimentary luminosity ratio. It is interesting to notice in the latter connection that the shape of the spectral "color-triangle" indicates a marked variation throughout the spectrum of the minimum objective purity necessary for undiminished subjective saturation, with a maximum in the blue-green region at or near the "peak" of the triangle. In the spectral stimuli used for the final observations a special investigation failed to show either a diminution in saturation or shift in hue as compared to sensibly monochromatic stimuli of the same mean wavelength. The average widths in terms of wavelength increments of the spectral bands constituting the stimuli are stated in the results.

The luminosities of the colored stimuli were determined in the larger part of the observations by a modification of the direct equality-of-brightness method which has been described by Pfund.[3] Smoke-glass wedges, the density gradients of which are vertical and are opposed in direction, are placed in the spectral and standard white beams (wedges not shown in Fig. 1). The photometer field then has the appearance shown in Fig. 1a. The observer's criterion for equality of luminosity is that of equal contrast on the upper and lower halves of the field, or of minimum distinctness of the dividing line at the mid-point. The introduction of the wedges appears to reduce very materially the difficulty of making a "heterochromatic" brightness comparison to an observer who has had no previous training. Pfund has found

that the luminosity values so obtained tally closely with the flicker-photometer. In taking observations, the observer is permitted to color-match the two halves of the field and to make luminosity determinations on the neutral mixture without use of the wedges, which are put in place only for the photometry of the separate colored components. The wedges are mounted in such a manner as to fit accurately into place, and, as proved necessary, the spectral wedge is accurately compensated with a wedge of clear glass to avoid deviation of the spectral beam when it is inserted.

Since the glass of which the wedges is composed is not perfectly non-selective its spectral transmission data had to be ascertained, after which the wedges were calibrated in place. The luminosity without wedges, L, of a spectral stimulus of mean wavelength λ could then be obtained in terms of the luminosity with wedges, L', by the formula $L=\frac{L'T}{T_\lambda}$ where T is the factor by which the luminosity of the standard white is reduced by the introduction of wedge, and T_λ is the transmission of the spectral wedge at wave-length λ.

PROCEDURE

A total of eight observers has been examined for complementary wave-lengths, of whom five furnished luminosity data. Of the latter, two had had much previous experience in color matching and one some experience in spectrophotometry. Three prospective observers were rejected in preliminary investigations on the grounds that the deviations of their individual settings were unduly great. In one case at least the trouble seemed to be in refractive defects, since the individual was unable to distinctly see the dividing line between the halves of the photometric field. One observer (the writer) furnished a complete set of data. The remaining observers furnished observations for only four pairs of complementaries, selected at about equal intervals on the color-triangle of König. One color of each pair was given in advance, the observer adjusting the wave-length and intensity of the other spectral stimulus, and the intensity of the standard white, so as to form an exact match. The intensity of the given stimulus was selected in such a manner as to demand an intensity of the neutral stimulus approximating to a certain predetermined value, of the order of 75 to 100 photons on the retina of the observer's eye. No attention was paid to the observer's state of dark-adaptation but it is probably true in every case that the previous environment had been a daylight illuminated interior.

Two of the chosen pairs of complementaries involved the terminal hues of the spectrum. The extreme red beginning about the Fraunhoffer line *C*, has been shown by Abney to be practically constant in hue, which same is indicated by L. A. Jones'[5] "scale of equal hue differences" for the spectrum. Accordingly the stimulus representing the extreme red was permitted to include nearly the entire region from line *C* to the end of the visible. At the other end of the spectrum, Jones' work shows continual alteration of hue out to about λ 400mμ. However, all hues from λ400 to about λ445 apparently have exactly the same complement. It is perhaps possible that the eyes of different individuals may vary in that respect but evidence to be adduced hereto indicates the contrary. Two stimuli composed respectively of wave-lengths centering about λ445 and λ400 could nevertheless be differentiated by their appearance. At first a difference in subjective saturation unaccompanied by change in hue was suspected, but by a determination of the complementary luminosity ratios of the two stimuli, which gave identical results for each, it was found not to occur. The dissimilarity of the visual impressions received in the two cases may be attributable in part to the rapid increase in the aberrations of the refractive system of the eye which takes place in the violet. In support of that hypothesis there is the opinion entertained by A. Ames, Jr., as a result of an extended study of the dioptrics of the eye, that chromatic aberration at the extremities of the spectrum especially in the violet is instrumental in enabling color-blind individuals to differentiate certain colors or colored objects.

The region from about λ445 on, therefore, was employed for the extreme violet stimulus. A blue filter was placed over the violet slit to eliminate stray light. The remaining selected stimuli were an orange λ 609.5, and a yellow λ 581.

A color-match having been obtained to the observer's satisfaction he then proceeded to determine the luminosities of the neutral mixture and each of its components separately in the manner outlined, several photometric settings for each luminosity being made without disturbing the adjustments pertaining to the colored stimuli. The luminosity of the mixture was ascertained as a check on the heterochromatic luminosities which should presumably summate to the same value.

RESULTS

The numerical results are comprised in Tables 1, 2, and 3. Table 1 contains the results of the detailed investigation of one of the observers. The first two columns give the mean wave-lengths of the comple-

mentary stimuli, the third column the luminosity ratio of the longer to the shorter wave-length, the fourth column the difference of the sum of the separate luminosities of the colored stimuli and the luminosity of the white mixture, expressed in percent. All luminosity determinations were made by the simple equality-of-brightness method without the use of wedges. The figures in the fourth column are in a general way a gauge of the accuracy of the luminosity measurements. The fact that the former are nearly always positive indicates that the effect of hue in the colored stimuli has been to increase apparent luminosity. Mention should be made of the fact that the comparison white of this set of data differs very slightly in quality from that for which the spectral distribution has been given, which latter was adopted for all other observations except as noted.

Table 2 contains all the wave-length data apart from that given in Table 1. In a few cases it was necessary or expedient to use a selected wavelength other than those specified at the heads of the columns, in which cases the selected wavelength is stated in parentheses beside the corresponding observed wavelength. Nearly all of the observed wavelength data are the averages of two or three settings. In many cases the deviations of the individual settings were less than the smallest division of the wave-length scale ($<1m\mu$). In the last three sets of observations recorded in the table, the quality of the comparison white was altered in divers ways, as indicated. Under the headings $\Delta\lambda_1$ and $\Delta\lambda_2$ are given the widths in millimicrons of wave-length increment of the spectral bands comprised in the stimuli corresponding to λ_1 and λ_2. The observer who furnished the data of Table 1 repeated observations both of wave-length and luminosity under the standardized conditions of Table 2 so as to obtain data comparable with that of the remaining observers, which data are included in Tables 2 and 3.

In Table 3 are given the luminosity ratios of longer to shorter complementary wave-lengths for five observers whose wave-length data appear in the preceding table. The given spectral stimulus of the complementary pair for which the different ratios apply is indicated at the top of each column. The figures in the last column are analogous to those of the column similarly headed in Table 1, in the present case, however, being arithmetical averages for all the observations of a stated observer. For purposes of comparison, data given by Helmholtz[6] are included in Table 3, which, apparently, are the only previously extant data of this sort obtained by direct observation. Al-

though in the table they are placed opposite the wave-length labels which apply to the rest of the data, Helmholtz designated the complementary pairs for which they hold merely by color names. He found that the values of the ratios were subject to considerable variation with change in the level of absolute luminosity at which observations were made, and gives two sets of figures corresponding to "low" and "high" absolute luminosity, of which the latter are here reproduced.

The luminosity ratios of complementaries may be obtained indirectly from "primary color" data, if taken in conjunction with the "visibility function" of the average eye (the personal visibility data supplied by König and Abney without much question depart farther from the fact than do the average data). It is known that wide variations occur in individual visibility characteristics, thus little is to be gained by a comparison of the results afforded by the individual color-triangles. The calculation has been carried out on the basis of the convenient average color-triangle reduced from the data of Abney,

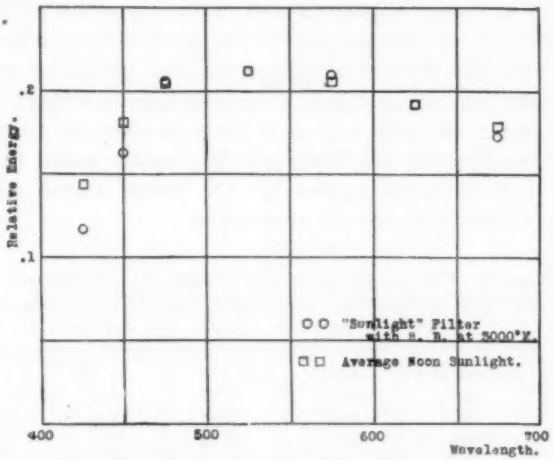

FIG. 2. *Spectral energy distribution of artificial source compared with that of average noon sunlight.*

König and Dieterici by Weaver, as published in the 1920-21 Report of the Committee on Colorimetry of the Optical Society of America,[1] together with other necessary data all taken from the same Report. The results are given in Table 3. Too much reliance should not be placed on the latter, however. The agreement between the color-data and the average visibility function (Cf. Tables 6 and 2 of the above

Report) is not as good as it should be, for if a different set of three points is employed to fix the luminosity coördinates of the color-plane, materially different values for the ratios may result. The points used in the calculation were: λ 450, 550, 660 mμ. Thus if λ 430 is taken instead of λ 450 the violet ratio is above 100.

In addition to these values obtained from the average color-triangle, the writer has also calculated values from the individual color-triangle of König. It was found that if the above wave lengths were used (with average visibility data for sunlight) König's violet elementary had to assume a definitely negative intrinsic luminosity, but if λ490 were used instead of λ 450, all the elementaries had positive luminosities and the ratio for the extreme violet came out about 6. The fact that this value departs so widely from that for the mean color-triangle makes the probable error of the latter extremely large.

It may also be mentioned that v. Frey and v. Kries,[7] as well as Angier and Trendelenberg[8] recorded slit-widths giving complementary amounts of spectral complementaries, with other data pertaining to the calculation of complementary energy-amounts, which latter together with the average visibility function would be capable of yielding luminosity ratios, but in both cases there have been omissions of certain important data such as to leave an element of uncertainty in the deduced results and since also the results would be without value for individual comparison for the reason stated above, the reduction of these data was not undertaken.

TABLE 1. *Wave Lengths and Luminosity Ratios of Complementaries*

λ_1	λ_2	L_1/L_2	L_1+L_2-L
Extreme Red	496.5	.73	2.2%
609.	493.5	1.10	3.8
591.	490.	2.03	3.5
586.	487.5	2.84	2.2
580.	482.5	4.8	2.4
578.5	480.5	5.9	1.9
576.5	477.5	7.2	0.5
575.5	474.5	9.2	−0.2
574.	472.	11.6	1.2
573.	466.5	16.0	1.4
572.	459.	24.6	1.7
570.5	Extreme Violet	40.	1.7

Observer R. H. S.

DISCUSSION OF RESULTS

1. DERIVATION OF NUMERICAL VALUES FOR THE SUBJECTIVE SATURATION OF SPECTRAL COLORS

Helmholtz,[6] who was the first to notice that complementary amounts of the spectral complementaries in general differ in luminosity, attrib-

TABLE 2. *Complementary Wave Lengths*

Observer	λ_1 (given)			
	Red	609.5 mμ	581	Violet
	λ_2 (observed)			
A H P	498 mμ	494	487	569.5
A E	500	495	479.5 (587)	582
R H S	495.5	493.5	484.5	569
H T W	503.5	500.5	491	569
B K	494	491	485*	569
H N E	489.5	487	480	566
P F G	495*	492*	487.5*	568.5
R	498	496(612)	488(583)	570
	Na Cr O_3 filter introduced.			
R H S	501	498.5	487.5	571
	Color-temperature of source increased. C T = 5000°+.			
R H S	495	492	483	569.5
	Sunlight filter removed. C T = 3000°.			
R H S	502	500	479	575.5
		$\Delta\lambda_1$		
		1 mμ	3	
		Mean $\Delta\lambda_2$		
	16	16	16	2

Starred observations excepted, all probable errors not greater than 1 mμ.

TABLE 3. *Luminosity Ratios of Complementaries*

Observer	Given λλ				L_1+L_2-L (average)
	Red	609.5	581	Violet	
	L_1/L_2				
A H P	.58	.96	3.3	26	4.2%
A E	.28	1.08		20.5	5
H T W	.64	.85	1.83	17.5	10.5
R H S	.67	1.0	3.8	36	6.1
H N E	.55			28.5	4.1
König's triangle	.36	.72	3.8	5.8	
Average triangle	.51	.96	6.4	82.5	
Helmholtz	.44	1.00	4.0	10.0	

uted the fact to a difference in subjective saturation among the spectral colors. From the values which he estimated for these luminosity ratios, together with the results of observations of similar character upon colored mixtures involving the central green, he was able to arrange the spectral colors in order according to their degree of subjective saturation, rating violet as the most saturated color, and yellow the least. He did not, however, attempt a numerical evaluation of the relative saturations, stating that he was constrained from so doing by the very considerable extent to which the values of the luminosity ratios of complementaries were subject to the influence of the Purkinje phenomenon at the level of absolute luminosity at which his experiments were carried out.

Very recently Troland[1] has investigated the quantitative relations in which the spectral colors stand in regard to saturation in a manner totally different. Troland noted that the spectral colors, if alternated with white in the flicker photometer, have different critical flicker frequencies. Assuming that the critical flicker-time of a color under these circumstances is in a way a measure of the extent to which it differs from white, and hence that colors equally saturated in the subjective sense ought to have equal flicker-times, he added white to each of a set of representative spectral colors in such amount as to give to each diluted color the same flicker-time, under which conditions the amounts of white which had to be added to the various colors served as a basis for estimating their relative saturations in the pure state. The outcome of the investigation is in general agreement with Helmholtz's conclusions, indicating that the maximum saturation is to be found at the violet end of the spectrum and that a minimum of saturation occurs in the yellow. The position of the minimum is fairly definitely localized at λ 575 mμ.

An idea of the relative saturation of the spectral hues may be arrived at from still another angle. In the treatise on the extension of the Weber-Fechner principle to apply to the elementary color-variables, the very great interest of which has perhaps escaped full recognition, Helmholtz[9] has plotted the spectral colors in what is probably nearly their correct relations to the elementary variables. The Helmholtz color-diagram, which is obtained from the color-triangle of König by a simple linear transformation of coordinates, has the unique property that near the center of the diagram a just perceptible hue-difference is represented by an equal distance at every point and in all directions. Thus the distance between the locus of any color and the white-point

at the center, if not too great, measures the number of just perceptible hue-steps between the color and white, in other words, measures its subjective saturation from a new standpoint. A glance at Helmholtz's diagram shows that the locus of the spectral colors, which falls near enough the center for the above relation to hold approximately, approaches the center in the yellow in the vicinity of λ 580, indicating a minimum of saturation at that point.

After a departure toward the periphery of the triangle in the blue-green the curve returns toward the center in the blue in which respect there is a failure to agree with the results of other methods of investigating saturation. One is scarcely justified, however, in discrediting other features of Helmholtz's investigation on that account. There is reason to suspect that König's color-triangle itself contains conspicuous errors in the region of blue and violet, which under the processes of Helmholtz's computations might conceivably be greatly magnified.

Some indication of the relative saturation of the spectral hues is also afforded by the average deviations of individual equality-of-brightness settings between spectral colors and white made in the process of the photometry of the spectral stimuli in the present investigation. These are found to be least in the yellow and greatest in the blue. Table 4 gives the mean values of the deviations of individual settings computed from the data of two observers.

TABLE 4. *Average Deviations of Photometric Settings*

λ	650	610	580	570	495	485
a.d.,%	2.5	3.5	1.5	1.0	3.0	5.2

All methods of investigation concur in showing a minimum of saturation in the yellow, probably at about λ 575 mμ. Use can be made of this information to correlate the present data for the luminosity ratios of complementaries in terms of the subjective saturations of the spectral colors, if it be postulated that the subjective saturations of two complementaries are inversely proportional to their luminosities.

It happens that the complements of practically the entire blue and violet regions of the spectrum are crowded into the region of rapid hue-change between λ 570 and λ 580. Since the saturation function has a minimum at about the middle of this interval and does not increase rapidly on either hand, it may be assumed without overstepping the limit of accuracy to which this investigation lays claim that saturation is constant throughout that interval; one may con-

veniently assign it a value of unity whereupon the saturations of all the colors whose complements lie in this region become simply the reciprocals of the luminosity ratios of the several complementary pairs and may be plotted immediately.

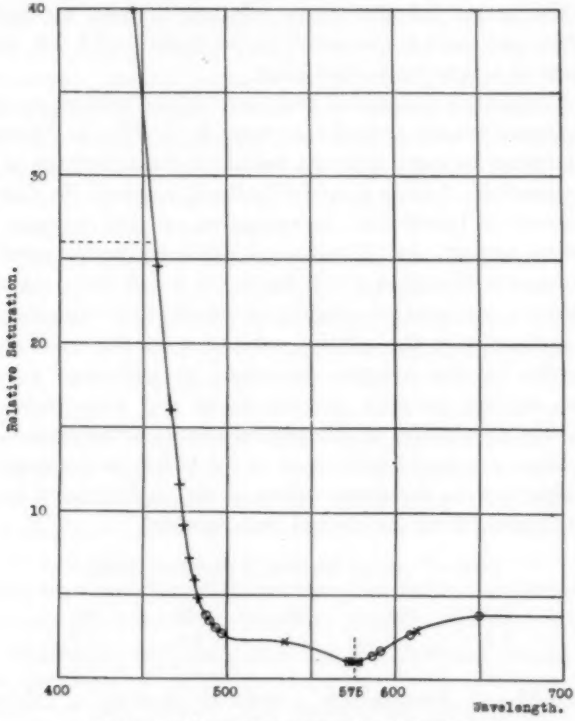

FIG. 3. *Relative subjective saturation of the spectral hues.*

Fig. 3 exhibits the data of Table 1 so plotted. Complementary pairs one member of which lies in the interval of λ570 to λ580, represented by the crosses, trace the saturation curve from the extreme violet to λ482.5, which is the complement of λ580. Here another region of rapid hue variation commences and all the complements of the remaining hues at the red end of the spectrum, from λ580 to the extremity, are contained in the brief interval from λ482.5 to λ496.5 which is the complement of the extreme red. A short extrapolation of the curve in the blue-green is thus sufficient to fix values for the saturations of all the hues beyond λ580. Enough points have been

taken in the blue to determine the exact course which the curve follows as it approaches the region to be extrapolated so that the extrapolation could be made with maximum assurance. In Fig. 3 the extrapolation has been carried out and corresponding points lying in the extrapolated region of the curve, and in the region complementary to it, are marked by circles.

In the gap between λ496.5 and λ570 a point has been tentatively plotted, from considerations of a similar nature to the foregoing. A cell containing a concentrated solution of $NaCrO_3$ was placed in the standard white beam of the color-matching apparatus thus producing an intense yellow on that side of the photometric field. This yellow proved to be nearly as saturated as the spectral yellow, and its dominant hue was readily determined by comparison with a monochromatic stimulus to be λ580. A spectral green and orange of λλ534 and 613 then were selected, on the principle that they lie at equal distances on either side of λ580, not on the wave-length scale, but on the scale of equal hue-differences of L. A. Jones. A mixture of these two stimuli was caused to match the chromate yellow, and the luminosity ratio of orange to green determined and found to be 0.8. Their respective saturations were assumed to be in the inverse ratio. The point λ613 lies on a portion of the curve already determined, so that the corresponding point can be plotted. In Fig. 3, the two points are marked by x's.

Having determined the form of the saturation curve based on the data of one observer, it may be readily modified to suit the average luminosity data of several observers contained in Table 3. No marked change will appear in the curve, except in the violet, where it will rise only to about the height shown by the dotted line, Fig. 3.

Comparing the present results with those of Troland,[1] it is seen that the latter fail to reveal the multifold increase in saturation in the short wave-lengths, although agreeing in their general aspects with the above. In view of the very different way in which saturation has been defined in the two cases the fact that different numerical results have been obtained is not as significant as the agreement in their general character.

It is possible that under different conditions a still greater value for the complementary luminosity ratio of the extreme violet would be obtained, more nearly agreeing with that afforded by the average color-triangle. Helmholtz found that the violet ratio was most altered by the presence of the Purkinje phenomenon, the complete absence

of which in the present investigation is not assured. Ives[10] sets as a safe upper limit to the range of the effect a retinal intensity of 75 photons for a luminous field confined to the fovea centralis. Troland[4] finds the limit for himself as an observer can be carried much lower, but suggests that individual differences in this respect may exist. In the present investigation the neutral comparison stimulus had an intensity above 75 photons, but the intensities of the spectral stimuli were necessarily lower, especially in the case of the violet, the photometry of which must have taken place at a level of three or four photons. The saturation obtained for the violet must therefore be looked upon as a lower limiting value which may be even doubled in the complete absence of the Purkinje effect. In fact, it was noticed that if in the process of finding the violet ratio, instead of taking the directly observed luminosity of the violet stimulus, a value for the latter is obtained indirectly by subtracting the luminosity of the complementary stimulus from that of the mixture, the ratio becomes as high as 90 or 100, indicating that the fraction of the luminosity of the mixture due to the violet component is nearly if not quite subliminal. This value, on the other hand, is probably too high, since the effect of hue in the complementary yellow stimulus would cause its luminosity to be overestimated—notwithstanding the fact that heterochromatic luminosity comparisons may be made with greatest precision at just this point in the spectrum—with the result that the difference value for the violet luminosity would be too small. The latter figure is thus an upper limiting value.

2. Individual Variations of the Color-Sense

Even among individuals whose perception of color is in no sense defective, variations in color-characteristics such as are evinced by individual differences in color-match equations are known to be considerable. Maxwell was first to suggest as a cause a variation in amount of the yellow pigment present on the macula lutea of the retina. Helmholtz considered that individual differences in color-characteristics are too great to be wholly traceable to that source. v. Kries and v. Frey[7] in the report of their joint investigation of spectral complementaries have shown that the discrepancies in their individual data are of a sort which a variation in the macular pigment would lead them to expect. In the discussion, they have plotted on the color-triangle the loci of complementary pairs and have indicated that interposition of a yellow tinted medium in the optical system of one

of them would induce variations qualitatively of the right sort to cause the two sets of data to coincide; moreover, that the existing variations are superficially similar in character to those found between the macular and extra-macular vision of a given observer. Hering[11] compared fresh mounted specimens of the human retina and verified the occurrence of marked differences in pigmentation among retinas from separate individuals. He also allowed "red-sensitive" persons to set up a color-match between a mixture of yellow and blue-green and a monochromatic intermediate green, and found the match held also for "blue-sensitive" persons when the latter viewed it through mounted retinas of the proper pigment densities. Hering entertained the opinion that variations due to pigmentation of the macula form a very considerable part, but not the entirety, of the total variability of the normal color-sense.

Complementary wave-length data afford a means of determining definitely the part played by the macular pigment in producing individual variations. Unfortunately previously extant data of that sort are ill-suited to comparison, because, as is well-known, the standard whites previously used have not been definitely enough specified as to quality, and have evidently varied considerably in separate investigations, while in no instance were more than two observers examined under uniform conditions. In the present investigation, as has been stated, eight observers were examined under identical experimental conditions. Individual differences in the results hence ought to rest solely upon actual differences in the color-characteristics of the individual.

The character of the individual differences can conveniently be exhibited by plotting the data on the color-triangle. It is a fundamental property of the color-triangle that the straight line joining any two given colors on the diagram is the locus of all hues produceable by their mixture. Accordingly, if on the color-triangle the loci of the spectral hues be known for a particular observer, and his complementary wave-lengths be joined by straight lines, all the lines will have the white-point of the diagram in common, and therefore all will intersect in a point. Now, if a second observer's complementary wave-lengths were plotted on the first observer's spectral locus, the lines joining corresponding pairs would not in general contain the white-point of the first observer. But if the sole cause of difference between their color-characteristics were variation in the pigmentation of the optical media of the eye, the second observer's several lines would still intersect in

a point, because under the conditions of experiment the selectivity of the optical media would be effective only in altering the hue of the white stimulus. The white-point of the second observer thus would be displaced from that of the first in the direction in which the quality of the standard white is modified by the addition or subtraction of the dominant hue of the pigment present in varying amount so that a straight line drawn through both points will cut the spectral locus at the dominant hue of the pigment, or its complement. Consequently, if one plots the data of several observers whose color-characteristics vary only from that cause all the white-points will be found to lie on a straight line. In the presence of other kinds of individual variations, the complementary data of one observer plotted on the spectral locus of a second, would not in general determine lines having a point in common.

The data of Table 2 of the present investigation have been plotted on the color-triangle of König. For this purpose, the color-triangle data contained in Table 26 (column *K*) of König's treatise entitled "Die Grundempfindungen in normalen und anomalen Farbensystemen" were plotted on a large sheet of coördinate paper. Points at λλ480, 490 and 500 mμ were interpolated from König's "Grundempfindungen" curves. As a further aid to plotting the complementary data, the distances between the plotted points were divided into a number of equal parts corresponding in each instance to the wave-length increment in millimicrons between the points.

Fig. 4 is a photographic reduction of the above diagram, on which have been plotted not only the data of the present investigation but also all other available complementary-wave-length data meriting consideration.* If all the lines joining the complementary pairs had been drawn in full, the diagram would be utterly confused. It is hoped that by tracing merely the polygon or other figure which each set of lines forms about the point the sum of the squares of whose distances from all the lines is least, and by using different styles of delineation, the reader will be enabled to differentiate the separate groups. The heavily inked configurations belong to the observers of the present investigation and may be associated with the corresponding data in Table 2 by the attached initials. The authorship of the remaining data is indicated in the legend accompanying the figure.

It may be seen at a glance that variations in the pigmentation of

*The only exceptions are the data of R. Schelske[13] and those of König's "anomalous" observers Zehnder and Becker,[18] which are omitted because of their paucity.

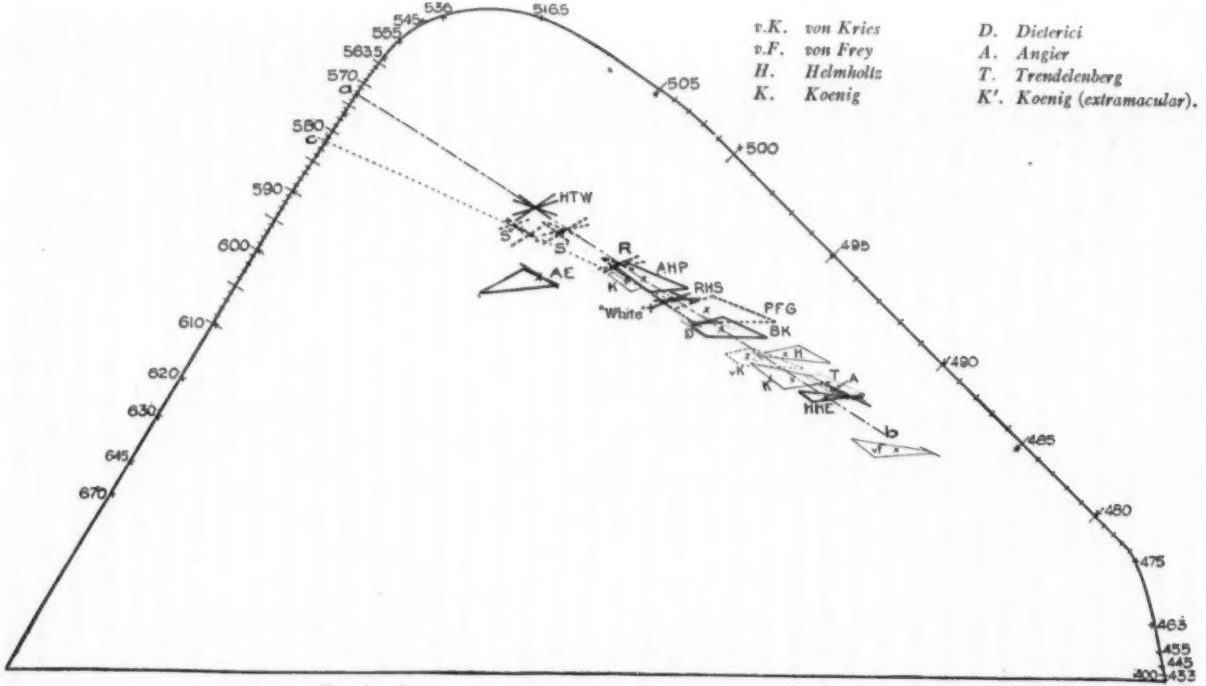

FIG. 4. *Complementary wave lengths plotted on the color-triangle of Koenig.*

the ocular transparent media are responsible for by far the greatest share of the variability depicted, and that the variation of the white point is very considerable. It is not, however, as great as might be inferred prima facie from the distances between the points on Fig. 4, because as is evident from the length of the wave-length divisions along the margin of the triangle the diagram is distended in a direction parallel to that along which the major variation takes place. Helmholtz's transformed coördinates would give better visual representation of the amount of hue variation corresponding to the variability of the white-point, but it was for the reason that the individual points are further separated on König's diagram that the latter was adhered to. On casual inspection it is also evident that the white-point variation, is due in the main to the addition or subtraction of one specific hue. Close examination reveals that the white-points, i.e. the points at the centers of the configurations formed by the different sets of lines, of six of the observers of the present investigation lie sensibly on a straight line, the line *ab* in the diagram. Observer (*P F G*) whose white-point in the figure departs perceptibly from the line *ab* made wave-length settings with distinctly lower precision than any of the other observers. The amount by which his white-point departs from the line is certainly within the limit of observational error. His data are of interest chiefly in that they show no exception to the general conclusions which can be drawn from the investigation. Observer (*AE*) is distinctly anomalous. The line *ab* cuts the spectral locus at $\lambda 572.5$ mμ, which may therefore be regarded as the dominant hue of the macular pigment, taken in conjunction with the spectral distribution adopted as standard in these investigations.

As an interesting test of the correctness of the considerations upon which the preceding argument is based, an attempt was made to modify the conditions of the experiment in a manner capable of imitating for a given observer the state of color-perception which would exist as a result of variation of the transmissive properties of his ocular media, of the sort which has been postulated. A thin cell containing a rather dilute solution of sodium chromate was found experimentally to possess in conjunction with the standard white illuminant a dominant hue very nearly that given by theory for the macular pigment. With the yellow filter placed in the standard white beam the writer obtained a set of complementary data, included in Table 2 under an appropriate label, which is shown plotted in Fig. 4 at *S'*. The result is, of course, what the theory of the color-triangle predicts. The hue of the chromate

solution is evidently very slightly toward red as compared to the macular yellow.

Two other sets of observations were also made by the writer, in which the color-temperature of the standard white was varied along approximately the black-body route. The primary object of the investigation was to discover how nearly the effects of such variation imitate the individual variations due to the macular pigment. One color-temperature, somewhat higher than the adopted standard, was gained by increasing the filament-current of the lamp which served as the source. The second abnormal color-temperature was obtained by removing the "sun-light" filter from the path of the light from the source, which was then operated at its normal filament-current corresponding to a c. t. of 3000°K. Both sets of data are included in Table 2 under suitable captions. The point determined by the first set of data falls so near that for the normal color-temperature of the same observer that it has not been plotted in Fig. 4. The second point is shown at *S''*. The line passing through this point and that for the normal color-temperature labelled (*RHS*) cuts the periphery of the diagram at *c*, corresponding to a hue-change in the standard white imitable by addition of λ581.5. Line *c* is of course the secant of the locus of the black-body series of hues, the tangent of which at the point where it intersects the "white" line would cut the spectral locus a trifle nearer *a*, but not near enough to be able to compensate for pigment-variations by mere changes in color-temperature of the standard white.

The white-points of observers not examined in the present investigation obviously cannot be expected to lie exactly along the line *ab* on account of non-conformity of the various standard whites. It is to be noticed, however, that they do without exception lie very close to the line, their mutual displacements in a direction parallel to the line being of a higher order of magnitude than the distances by which they depart from it. In every case some species of daylight was used as the standard white illuminant. The variations in the standard whites are probably greater than the departures from the line might seem to show, however, since the larger part of the variation would likely occur in the direction from yellow to blue, hence, parallel to the line.

It may be noticed, moreover, in cases of investigators who have worked in pairs, that the white-points lie on lines nearly parallel to *ab*. Angier and Trendelenberg have points so nearly coincident that conclusions can scarcely be drawn but the fact is well brought out

by the data of von Kries and von Frey, and König and Dieterici. The latter pair present a particularly interesting case. Not to add to the complexity of Fig. 4 by drawing another line, the data of the last-named individuals have been plotted separately in Fig. 5 in a different fashion, which illustrates very clearly the point in question. Fig. 5 is a copy of "Fig. 4" of König's paper "Die Grundempfindungen in normalen und anomalen Farben Systemen"[13] the data for which is

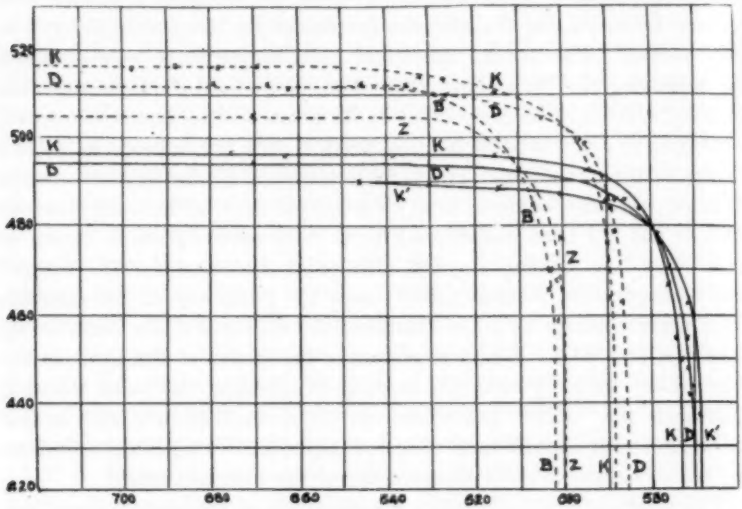

FIG. 5

contained in Tables 8 and 9 of the same treatise. The solid curves marked *K* and *D* represent the complementary wave-lengths of König and Dieterici with sunlight as the standard white illuminant, with longer wave-lengths as abscissas and shorter ditto as ordinates. They may be observed to intersect at about λλ578-477. The third solid curve marked *K'* is added to the curves of the original diagram from certain data given out by König several years later,[14] which data specify the author's complementaries at a point on the retina about 3° below the normal fixation point, hence in extra-macular territory. A secondary source was here used for the standard white, but was very carefully matched to sunlight according to the author's statement. Apparently the later investigation was entirely independent of the first. König does not make use of the wave-length data obtained in the later investigation.

A glance at Fig. 5 of the present paper shows that curve K' intersects curves K and D almost precisely at their common point. This is of course equivalent to the circumstance that the "white-points" on the color triangle corresponding to the three sets of data should lie on a line. In Fig. 4 of the present paper the point K' corresponds to curve K' of Fig. 5. It may readily be seen that the three points considered do lie on a line altho the line has not been drawn. The principal differences which exist between the color-vision of these two individuals are therefore of exactly the same type that exist between the macular and extra-macular vision of one of them, and hence must be concluded to be due to variations in the amount of the macular pigment.

This conclusion may be still further supported from the following considerations: Sachs[15] measured the spectrophotometric transmissions of freshly prepared specimens of the human retina from nine individuals. König[16] calculated mean figures for the transmission of the macula from Sachs' data. Ives[17] recently outlined a method of determining the dominant hue of a selective substance, given its coefficients of selectivity and König's "sensation" curves. Following this procedure, the dominant hue of the macular pigment with König's sunlight as the illuminant turned out $\lambda 579.5$ mμ, in almost unwarranted agreement with $\lambda 578$ as the intersection-point of the curves of Fig. 5. Dominant hue is of course a function of the spectral distribution of the illuminant, so that it is perhaps not to be expected that the value given for the macular pigment by König's data would coincide exactly with that given by the results of the present investigation. The fact that the two agree as nearly as they do leaves no doubt that in both cases the same pigment has played the rôle of the variable factor.

There is, however, an occasional individual who altho in no degree color-blind is clearly anomalous. One such was encountered in the present investigation in the person of observer (AE). His individual observations show no greater or at any rate not significantly greater probable error than do those of the majority of the observers, which is proof that he is not color-defective. The amount by which his white-point deviates from the line of normal variation is in terms of hue-difference relatively very considerable. It is again to be concluded, however, that the principal factor in the variation is a selective modification of the radiant energy reaching the retina, perhaps due to the presence of some unusual coloring matter in the media of the eye rather than to variation in amount of some pigment already present

in the normal eye since the other observers show no tendency toward a two-dimensional variability. The possibility that the unusual pigment was of pathological origin appears to be ruled out by the circumstance that the observer's color-vision is alike in both eyes.

Cases of "anomalous" three-dimensional or "trichromatic" color-perception have been reported in the literature. König[13] obtained complementary data from two observers whom he styles "anomalous." These data are shown plotted in Fig. 5 and determine the dotted curves *B* and *Z*. In the original figure from which Fig. 5 is copied only the lower portions of these curves are drawn. In the accompanying table in König's paper a few additional points are given which have been used to fill in the rest of the curves. The white illuminant was in this case "gas-light."

For the sake of comparison, König obtained also sets of complementary data for himself and Dieterici with "gaslight" as the white illuminant, and these determine the dotted curves *K* and *D* in Fig. 5. König points to the evident grouping of the curves into distinct sets as proof that the former observers are "anomalous." Note, however, that when their curves are extended they are found to bear about the same relation to one another as those of König and Dieterici, including an intersection in about the same relative location, thus the "anomalous" observers closely resemble one another and their mutual differences are apparently of the same type as those of the normal observers *K* and *D*.

Both observers *Z* and *B* were also subjected by König to a complete three-color analysis. A comparison of these latter data with the similar data of *K* and *D* shows no such discrepancies. The "Elementarempfindung" curves of *Z* have been plotted by König together with those of *K* and *D*, from which it is evident that the complements of the extreme hues of the spectrum, given by the points of intersection of the curves, differ less for *D* and *Z* than for *D* and *K* and what difference exists is in the opposite sense from that shown by the curves reproduced in Fig. 5.

Apparently no actual abnormality with respect to color-characteristics existed in these cases.* More probably, the differences exhibited in Fig. 5 are due to a variation in the quality of the "gas-light" illuminant. Altho there is no definite statement to that effect, it seems probable from the context that the two pairs of observers were examined

*N. B. König omits to state on what basis these observers were originally classified as anomalous. Probably a test depending on the form of the visibility curve was used.

on separate dates. A description of the gas-burner is also wanting. The color of a gas flame is known to vary considerably with a difference in the burning conditions, or in the composition of the gas. König's specific assertions as to the abnormality of these observers are not supported by his data.

As to the cases of "anomalous" color-perception which have been reported by other investigators no definite conclusions can be drawn, because the tests applied to them have concerned merely the form of the "visibility" function of the individual, which need not necessarily bear any relation to his color-characteristics. It is to be noted that among the fifteen observers whose data are plotted in Fig. 4 certainly no marked abnormality occurs except in the single case which has been discussed. The standard white variations superposed upon the characteristic ones prevent an absolute statement.

Besides the major variations affecting the location of the white point, Fig. 4 shows in most cases a failure of the groups of complementary lines to intersect precisely in a point. One notices, however, that the configurations of the separate sets bear a certain resemblance. In particular this is true of the portions of the figures formed by the lines joining the blue-yellow and the violet-yellow-green complementary pairs, which display a divergence toward the blue corner of the diagram, that is more marked in the figures lying nearer the blue. This is true without exception, even of the configurations formed by König's own data at K and K'. Apparent exceptions are the figures of A and T, but these investigators did not extend their complimentary observations into the extreme violet, so that the complementary pairs represented in the figures are not the same as in the other cases. In other cases complementary pairs corresponding approximately to those selected for the present investigation were plotted so as to have the results in the most comparable form.

Thus it is evident that alteration in the color-triangle in the region of the blue or violet would materially reduce the total area included by the small figures, hence one may conclude that the triangle is in error in this region.

On the other hand as regards the pairs of lines ending in the blue-green, which cut the former pairs more or less transversely, the apparent discrepancies are greatly magnified by the distention of the diagram approximately at right angles to them. In no case would a change in the blue-green member of a complementary pair greater than one millimicron be needed to cause the corresponding line to pass through

the respective white-point marked in the diagram. Taking into account possible small errors in the color-triangle in the blue-green or red regions together with the probable observational error of the complementary determinations, which is greatest of all for the pair including the extreme red, the failures to intersect are certainly within the limit of error of the investigation. The configuration enclosing the greatest area is that of (*PFG*) whose data as previously stated also have the greatest probable error in the present investigation.

The conclusion has therefore been reached that the results of the present and previous investigations fail to reveal inherent variations in the complementary wave-lengths of separate individuals other than can be accounted for by differences in the absorptive properties of the ocular media, and that if variations of other types occur, they are so small as to be barely detectable through the greatest attainable accuracy in color-matching.

The situation seems sufficiently general to warrant the use of the complementary data as criteria for testing the accuracy of the various extant color-triangles. A few representative sets were plotted on Maxwell's two triangles and that of Dieterici as well as that constructed from the average data of König, Dieterici and Abney by Weaver.[1] Space does not permit publishing the diagrams. The triangle of König is somewhat better than the rest, including the average triangle, according to this standard.

The data of the present investigation determine a mean white-point for König's triangle at about the location of the white-point of observer (*RHS*). The original white-point of the triangle, shown in Fig. 4, happens to lie very near here.

Relative to a possible relationship between the amount of pigment present on the macula and on the iris, it was noticed that the only blue-eyed observers (*HTW*) and (*AE*), show the greatest amount of absorption in the transparent media; but the relation, if any exists, is evidently not close because some of the observers whose white-points lie near the middle of the range have very dark eyes, while the iris of the observer with least macular pigment is light brown or "hazel."

The amount of macular pigmentation of the left and right eyes of the same individual appears to be identical in general. The writer is able to detect qualitatively a barely perceptible difference in his own eyes. One or two other observers seemed to notice a difference slightly more marked.

The question now suggests itself as to whether the individual "visibility" function is invariant if correction is made for absorption in the ocular media. The question is not answered by the present investigation as satisfactorily as is to be wished; apparently, however, the case is otherwise.

The present and other experimental investigations show that luminosities are distributed on the plane of the color-triangle very nearly according to the same law that applies to hue. Therefore to every point in the color-plane in addition to the two coördinates describing its hue can be attached a third coördinate proportional to its luminosity thus completely fixing its position in the color-space. The knowledge of the three coördinates of three points not in a straight line is sufficient to determine the plane, and therefore to fix the luminosity coördinates of all points on it. That fact has already been used in finding the luminosity ratios of complementaries from the color-triangle. The assumption of an invariant visibility function is equivalent to assuming that the color-plane has identical luminosity coördinates for all individuals.

Turning to Fig. 4, it appears that the pair of spectral hues given by the intersections of the line *ab* with the spectral locus are complementary for all the observers whose white-points lie along it. Supposing the luminosities of these spectral hues to be invariant on the color-plane, mixtures of them which are complementary to the different observers would have luminosity ratios dependent in a definite manner upon the locations of the different observer's white-points. The luminosity ratio of longer to shorter wave-length would be greater corresponding to white-points lying nearer the longer wave-length hue. The exact relation in which the luminosity ratio should stand in the different cases can be worked out from the diagram.

The universal pair of complementaries was not one for which the luminosity ratios were experimentally determined. However, the pair including the terminal violet is practically invariant, showing perceptible variation only in the extreme case of (*HNE*). The luminosity ratios of this pair given in Table 3 do not exhibit variations of the sort anticipated from the initial postulate of an invariant visibility function. The ratio of (*HTW*) which on that hypothesis should be greatest, is actually least and is less than half that of one of the other observers, while that of (*HNE*) which should be least, is next to the highest.

These differences are certainly larger than the probable errors given by the deviations of independent settings. It is possible, however, that the results are influenced by the Purkinje effect more in the case of some observers than of others. A general discussion of the Purkinje effect in connection with the observations was entered into in the preceding section. It has been suggested that structural differences of the retina may exist which would render some individuals liable to the Purkinje effect on the foveal area, while others are free from it. From this circumstance, the indications of inherent variations of the visibility function are regarded as inconclusive.

No more definite conclusions can be drawn from a comparison of the luminosity data applying to other complementary pairs which might be effected thru the medium of König's color-triangle, because of discrepancies between the color-triangle and the data of any single individual of the order of magnitude of those existing between different individuals.

In an effort to dispose of the question, some additional experiments were undertaken. It had already been found that sodium chromate in aqueous solution in conjunction with the adopted standard white illuminant has very nearly the same dominant hue as the macular pigment. A gelatine wedge impregnated with sodium chromate was prepared and inserted immediately in front of the ocular slit of the color-mixing apparatus thru slots cut in opposite sides of the tube on which the slit is mounted, which permit sliding the wedge back and forth thus altering the thickness of the portion of the dyed gelatine film covering the slit. The effect of the wedge as far as concerns the results of observations is almost precisely the same as altering the amount of macular pigment present in the eye of the observer. Obviously the question of whether the spectral transmission of the wedge coincides with that of the macular pigment is immaterial under the circumstances, the only factor of importance being its dominant hue in conjunction with the standard white. Observations made with the wedge show that the dominant hue of the latter agrees with that of the macular pigment even more nearly than predicted by the data obtained with the sodium chromate solution, in fact the agreement is so close as to be nearly if not quite within the limit of observational error.

Inserting the wedge therefore has the effect of varying the hue of the standard white very nearly along the line *ab* on Fig. 5, for any observer whose white-point originally lies on *ab*. By suitable adjustment of the wedge it is thus possible to cause the "white-point" of an observer

with a smaller amount of macular pigment to coincide with that of an observer with a greater amount. With the wedge so adjusted a set of luminosity data can be obtained by one observer which should be identical with that of another provided the visibility functions of the two observers are identical disregarding the macular pigment.

Unfortunately at the time the wedge was in use most of the observers who took part in the earlier part of the investigation were not available. Only one set of data was obtained with the wedge. They are given in Table 5 together with those of one of the other observers (taken from preceding tables) to which the former are supposed to be comparable. The two sets of wave-length data are so nearly identical that no attempt has been made to plot on Fig. 5 the set obtained with the wedge. It may be seen that the luminosity ratios for the extreme red and its complement are identical for the two observers under these conditions; discrepancies are present, however, in the cases of the other pairs of complementaries which are greater than are allowed by the probable error of the results given by the deviations of independent settings. Probably the Purkinje effect could not here be looked upon as the source of the discrepancies but whether they may be assigned to mere differences of individual judgment in making equality-of-brightness settings involving such wide hue contrasts may still be open to question.

TABLE 5

λ_1 (given)	λ_2 (observed)		Luminosity ratio	
	HTW	RHS +wedge	HTW	RHS +wedge
Red	503.5	503.5	.64	.64
609.5	500.5	500.5	.85	1.00
581	491	491	1.83	4.1
Violet	569	570.5	17.5	>40

The mathematical expressions for the most general relations which may exist among the elementary color-variables x_1, y_1, z_1, and x_2, y_2, z_2 of two individuals whose color-match equations, after having been corrected for absorption in the ocular media, are identical, take the linear homogeneous form:

$$\begin{aligned} x_2 &= a_1x_1+b_1y_1+c_1z_1 \\ y_2 &= a_2x_1+b_2y_1+c_2z_1 \\ z_2 &= a_3x_1+b_3y_1+c_3z_1 \end{aligned} \qquad (1)$$

where a, b, c are positive constants.

The simplest form of equations (1) compatible with invariant color-match equations but a varying visibility function is

$$\begin{aligned} x_2 &= ax_1 \\ y_2 &= by_1 \\ z_2 &= cz_1 \end{aligned} \tag{1a}$$

which would be realized in the case of three photo-chemical substances possessing definite response functions $x(\lambda)$, $y(\lambda)$, $z(\lambda)$, present in different relative amounts on the retinae of different individuals.

A consideration of the general form of the equations shows that a third type of individual variations may occur, apart from those which would be detected by color-matching or by luminosity measurements.

It is possible and in fact highly probable that the wave-length threshold of hue-difference discrimination in the spectrum is some function of the elementary variables x, y, z, and their derivatives with respect to the wave-length, $\frac{dx}{d\lambda}, \frac{dy}{d\lambda}, \frac{dz}{d\lambda}$. Thus Helmholtz assumed, altho it is not essential to do so, a function the form of which is a consequence of the Weber-Fechner principle applied in its unmodified state to the elementary variables. Differentiating equations (1) with respect to λ gives:

$$\frac{dx_2}{d\lambda} = a_1\frac{dx_1}{d\lambda} + b_1\frac{dy_1}{d\lambda} + c_1\frac{dz_1}{d\lambda},$$

etc.

By suitably choosing the undetermined coefficients of equations (1) it is obviously possible to assign arbitrary values to $\frac{dx_2}{d\lambda}, \frac{dy_2}{d\lambda}, \frac{dz_2}{d\lambda}$, as well as to x_2, y_2, z_2, at any one point where x_1, y_1, z_1, $\frac{dx_1}{d\lambda}, \frac{dy_1}{d\lambda}, \frac{dz_1}{d\lambda}$, are given; hence in general the function expressing the rapidity of hue change in the spectrum will have different values in the two systems of variables, and different distributions of hue in the spectrum would ensue. Only six of the nine undetermined coefficients of equations (1) are used up in assigning arbitrary values to the six variables, so that there still remain at one's disposal the three necessary to allow of an arbitrary fixing of the ordinates of the visibility curve. Variations of the last type may be seen to be distinct from the first two types discussed.

A comparison of data of several individuals is afforded by Steindler's[18] investigation of wave-length thresholds of hue difference discrimination. Steindler's data exhibit marked variations of a fundamental

character which would almost certainly exceed in magnitude observational errors under ideal experimental conditions. Unfortunately the report of Steindler's investigation is not sufficiently definite as to details of experimental procedure to permit of an appraisal of the claims to precision of the work it covers. It is difficult to conceive of an objective mechanism likely to give rise to variations of the last type without violating the conditions set forth in equations (1).

In conclusion the writer wishes to express his thanks to Dr. A. H. Pfund for several most useful suggestions and criticisms, and to all who kindly assisted in making observations.

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DEPARTMENT OF PHYSICS,
JOHNS HOPKINS UNIVERSITY,
BALTIMORE, MARYLAND,
JUNE 1, 1923.

Series in the Arc Spectrum of Gold.—The spectrum was photographed with a large quartz spectrograph, using gold in the bored-out positive electrode of a 220 volt carbon arc. Five members each have been found for a sharp and a diffuse doublet series, which are perfectly regular as regards intensities and inner quanta. The 1st pair of the principal series is at λ2428 and λ2676; the next pair would lie at about 1600 A.U.

$1s = 74510$; $2p_1 = 33242$; $2p_2 = 37058$; $\Delta 2p = 3816$;

the *md* separations are 82, 43, 39, 37, and 35, for $m = 3$ to 7, respectively. This slow decrease as the term number increases is unusual. It is scarcely possible that the difficulty can be due to incorrect wave length determinations. A similar phenomenon is observed in the so-called diffuse triplets of selenium and the diffuse series of Al and Al^+. [V. Thorsen, (Copenhagen); Naturwissenschaften, *11*, p. 500; 1923.]

ARTHUR E. RUARK

Phosphorescence of Solid Solutions of CdS in ZnS.—These solid solutions are isomorphous with ZnS in one of its crystalline forms (wurtzite) and with CdS; the phosphorescence is due to a minute admixture of a metal, in these cases copper. The phosphorescence-spectrum is a band with its maximum in the green (near 5400 A) for ZnS, in the yellow (near 5800) for ZnS with 20% CdS, and further towards the red (near 6000) for ZnS with 30% CdS. The visual color changes correspondingly, from bluish green to carmine red. The author thinks that these differences result from the absorption of the phosphorescence light on its way out through the crystal; a study of the absorption spectrum showed that the shorter waves are absorbed up to a threshold wave length which moves towards longer waves as the proportion of CdS in the mixture is increased. The admixture of even a small proportion of CdS prevents the formation of crystals of a second, less durably phosphorescent, crystalline modification of ZnS.—[A. A. Guntz; C. R. *177*, pp. 479-482; 1923.]

K. K. DARROW

INSTRUMENT SECTION

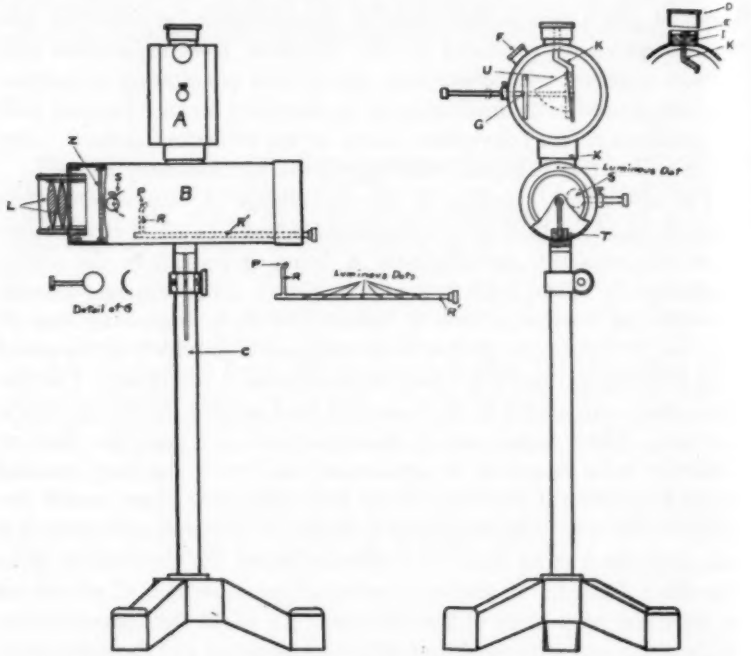
AN INSTRUMENT FOR SHOWING SOME OF THE PHENOMENA OF RADIOACTIVITY

By J. E. Underwood

Some of the phenomena of the spontaneously emitted radiations from radio-active bodies can be visualized in a variety of ways, and the instrument to be described was devised in an endeavor to produce a simple apparatus which would show some of the most important manifestations of these radiations. With it one is able to demonstrate some of the characteristic electric, photographic, scintillating and fluorescent effects produced by the radiations from radioactive substances together with absorption effects and penetrating properties. Radium is used in the instrument to be described because together with its products it is a convenient source of the radiations desired. Any other radioactive substance fulfilling these conditions may be used.

The instrument consists of an electroscope *A*, superposed upon tube *B*, which in turn is on an adjustable stand *C*. The electroscope is of low capacity, the electrode *K* being imbedded in the amber insulation *I*, which holds it firmly in place. The electrode extends through the insulation and is charged at *E* by removing cap *D*. The leaf *U* is either of gold or aluminum and is provided with a guard *G* to hold the leaf in place when the instrument is not in use. Circular glass plates are placed in the front and back so that the leaf system is inclosed. These plates are of clear glass with the exception that an arbitrary scale is etched on the front plate, with the lines running parallel to different positions of the leaf. The clear plates enable the image of the leaf to be thrown on a screen, or if a projection lantern is not available a small light hung directly behind the electroscope gives the effect desired. In the latter mode of operation it is of advantage to have the back plate of frosted glass. To avoid the contamination of the electroscope by the diffusion of the emanation and the subsequent deposition of the decay products, the electroscope is removed from tube *B*, at telescopic connection *X*, immediately after showing the ionization effects. As a further precaution against contamination, guard *G* is pushed against the leaf, cap *F* is removed and a current of dry air is drawn through the electroscope. Both sides of the telescopic connection *X* are provided with caps, which are put in place when the electroscope is removed from tube *B*.

Tube *B* contains the radium which is placed at point *P* at the top of rod *R*. The top of rod *R* is cut at an angle of 45° and the angle face is set toward the front of tube *B*. Rod *R* is perpendicular to rod *R'*, which moves in grooved track *T*, whose sides hold rod *R* in an upright position so that point *P* is held in the center of tube *B*. By moving *R'* the radium may be brought close to or pulled away from the glass screen *Z*, which is in focus with lens *L*. *S* is a cup shaped disc of thin aluminum, which is attached to a rectangular rod which moves in a rectangular groove. This allows the disc to be pulled close to the side or pushed into the center of tube *B*.



All of the visible effects, with the exception of the ionization effect, must be viewed in the absence of light and consequently after the electroscope has been used, all lights must be eliminated so that the eyes may become accustomed to the darkness. It goes without saying that any photographic experiments with the radiations must be done also in the absence of light.

Ionization effects:—Before placing the electroscope on tube *B*, a

"natural leak" is taken by imparting a charge to the leaf system and allowing the leaf to float down and noting the time interval for a given range. After the electroscope is place on the tube and charged, the discharge of the instrument begins immediately. The rate of fall of the leaf depends upon the position of the radium point along track *T*. The ionization is relatively the same at any point, but the number of ions reaching the leaf system is greatest when the radium is directly under the opening into the electroscope. This ionization is primarily due to the α rays, but by placing a thin sheet of paper over the opening on tube *B* before the electroscope is connected the α rays are absorbed. Then the ionizing effect is due primarily to the β radiation. The electroscope is not sensitive enough to show the ionization produced by γ rays because of the small amount of radium, (about 0.1 mgr), which is present in the instrument, excepting over a relatively longer period of time.

Photographic effects:—In the absence of light a piece of photographic paper or plate placed over the opening at *X*, will, after being developed, show the effects of the radiations. The effect of the β and γ rays can be shown by screening off the α rays as with the electroscope, or the effect of the γ rays alone by screening off both the α and β rays by interposing a lead plate at least 2 mm in thickness.

Scintillating and fluorescent effects:—These effects can only be seen to advantage when the eyes have become well accustomed to darkness. A small dot of radium luminous compound placed directly over the eye-piece of the lens *L* is of such low intensity that when it can be seen the effects within the tube are also visible. The screen *Z* is coated with phosphorescent zinc sulphide, willimite or barium-platino cyanide or a mixture of these or any other suitable compounds and they are placed on the side of the screen *toward* the radium. As in the case of the electroscope the amount of radium is so small that no very satisfactory effects can be produced by the γ rays alone. However the differentiation between the α and β rays can be clearly shown.

Absorption effects:—Rod *R* is calibrated to show the position of the radium in reference to the screen *Z*, by having each cm indicated with a luminous dot. When the radium is farthest from the screen, all of the α rays are absorbed by the air space. Under these conditions, if the screen is covered with phosphorescent zinc sulphide, no scintillations are seen through the lens, excepting those produced from the active deposit. These are, however, very few in number. When the radium is brought slowly toward the screen a position is reached where the

α rays are no longer absorbed by the air space and the scintillations instantly begin. This effect of countless flashes of light is intensified by moving the point closer and closer to the screen, and is the same as that produced in the spinthariscope as devised by Sir. Wm. Crookes. Many shadow effects are shown by moving the aluminum disc S. Pulled against the side of B it does not interfere with the passage of the radiations from the radium, but by pushing it into the center of the tube it absorbs the α rays coming in contact with it. By having the disc in the center of the tube and bringing the radium point close enough to have the zinc sulphide just within the range of the α particles, a circle of scintillations is seen which is intensified by moving the radium forward. When, however, the radium is moved into the center of curvature of the disc, the α rays are completely absorbed. This gives an effect analogous to complete absorption by air. Phosphorescent zinc sulphide is also luminous under the action of the β rays, but as the luminosity produced is relatively small to that produced by the α rays, it is reduced to a minimum by the small seat of radiation present. If the screen Z is coated with willimite or barium-platinocyanide, a fluorescent effect is noted even after the radium point is moved behind the aluminum disc. This fluorescence, which is due to the β and γ radiation, is shown to better advantage in differentiating it from any effect of the α rays, by having the screen Z coated with a mixture of phosphorescent zinc sulphide and willimite, (or barium-platino cyanide). The fluorescent effect of the direct radiation on the willimite, (or barium-platino cyanide), is small in comparison with the scintillating effect produced on the zinc sulphide by the α rays, but when the disc S is allowed to absorb the α rays the fluorescent effect produced by the unabsorbed β and γ rays is clearly seen. The compounds used on screen Z are all affected by the continuous bombardment of the radiations in that they become less sensitive. As a precautionary measure against this the aluminum disc is either pushed into tube B, or the radium point is drawn to the back of the tube, when the instrument is not in use.

Some of the effects enumerated as being produced in the instrument are roughly indicative of the methods used for the estimation of radioactive substances. The action of the rays on a photographic plate was used extensively in the earlier development of radioactivity. The ionization produced by the radiations on the surrounding gases has led to the electrical methods of measurement, which have superseded all other methods for the quantitative determination of radio-

active substances because of the rapidity and accuracy possible. Although the luminosity produced on willimite or barium-platino cyanide is only of use qualitatively, the scintillations produced on phosphorescent zinc sulphide by the α rays, has played a very important rôle in the recent researches into the physical nature of matter.

All of the phenomena as produced by the radiations from radioactive substances are important in characterizing them, but it is not possible to produce these effects except by the use of a variety of instruments, or one of great complexity. The instrument, as described, is simple in construction and operation and consequently many important effects are not shown. However the effects produced are typical of those produced by radioactive substances and which makes possible their distinction from all other forms of matter.

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NEW YORK CITY.

The Distribution of Intensity in the Broadened Balmer Lines of Hydrogen.—The lines H_β, H_γ, and H_δ, excited by a condensed discharge through pure dry hydrogen at pressures of 48, 112, 160, and 250 mm, and hydrogen mixed with helium and with nitrogen, were passed through a neutral wedge, dispersed by a prism spectrograph and photographed. The images of lines as reproduced with the text are smoothly conical, but on the original plates there were traces of structure; "H_β was noticeably a close hazy doublet at the apex, H_γ, and H_δ gave evidences of slight wings symmetrically placed on each side; and H_β exhibited a bright central core which was lacking in the case of H_δ." The Stark effect resolution of the three lines, when averaged over all values of the angle between line of sight and direction of field, should give pronounced "wings" in all three cases, with a more or less obvious minimum of intensity in the center; provided, however, that electric fields of a certain intermediate strength are much more frequent than either very strong or very weak fields. Hulburt calculates the distribution of intensity to be expected if (a) the field produced by one atom at the point occupied by another is inversely as the square of the distance between them; (b) the distribution of atoms is given by classical kinetic theory formulae; (c) the line is split into two symmetrically placed components. This distribution function has the specified form. It is to be supposed that only a small fraction of the atoms are either centers of force or sources of emission (whether we assume that they are the same atoms or different ones is irrelevant); but no attempt is made to attain a quantitative agreement, nor is allowance made in the formulae for overlapping fields. The broadening in the condensed discharge and the extra broadening when a quenched gap is inserted in series with the discharge tube, are ascribed to the high current density

which makes ionized atoms, acting as centers of force, more numerous; the narrowing produced by added inductance, to the lowering of the oscillation frequency and hence of the current density. Photographs of the broadening of the Balmer lines due to admixed nitrogen and helium are also given. [E. O. Hulburt, (Iowa) Phys. Rev. *22*, pp. 24-36; 1923.]

K. K. DARROW

X-ray Absorption-spectrum of Xenon.—The x-ray absorption-spectrum of gaseous xenon, enclosed at atmospheric pressure in a tube with windows of cellophane, was photographed by a conventional method. The three L absorption edges were measured; they are at 25.875, 24.253 and 22.724×10^{-9}cm. respectively, and lie on the same smooth curves as the corresponding values for the elements of the preceding period (the one extending from silver to iodine); but there are discontinuities both of the curves and of their slopes between xenon and the next following elements, indicating that just beyond xenon a rearrangement of the inward electron orbits takes place. The L_1 band appears to be somewhat broader than it would be if the breadth were due entirely to electrons being displaced from the L_1 orbits to virtual orbits of the valence electron lying beyond the last stable orbit (13 volts observed versus the 9 volts of the ionization-potential). The absorption-spectra of Cs^+(halogen salt of Cs) and I^- (iodide of an alkali metal) were also observed; the former displays two absorption lines near (at?) the expected positions of the L_2 and L_1 absorption-edges, which are supposed to result from electrons being displaced to the P_3 orbit from the L_1 and L_2 orbits; the latter shows faint absorption lines, resulting from electrons being displaced from the L-orbits into virtual orbits in the periphery of the atom. The authors seem also to have rediscovered the spurious absorption lines observed by Overn (Phys. Rev. (2) *18*, 350-355; 1923)—[A. Lepape & A. Dauvillier; C. R. *177*, pp. 34-37; 1923.]

K. K. DARROW

Spark Lines of Copper Visible in the Spark Spectra of Copper-Aluminium Alloys.—Investigation of the wave length range between 2170 and 4800 A, with alloys containing .005 to 5% copper, with tables of the lines visible between 2170 and 3280 for seven such alloys. The most persistent lines are those which persist in spite of the introduction of self-inductance into the sparking circuit. The entire spectrum of copper does not appear until there is 60% of copper in the alloy; the entire spectrum of aluminium appears at 20% aluminium. The copper lines disappear sooner with a Cu-Zn alloy than with a Cu-Al alloy. [X. Wache; C. R. *177*, pp 39-41; 1923.]

K. K. DARROW

NEW PROCEDURE IN TESTS OF AUDITORY ACUITY

By C. C. Bunch

Various types of electrical apparatus are rapidly replacing the tuning forks as sources of sound for tests of auditory acuity because of the ease, accuracy and constancy with which the results may be secured. The recent development and practical application of the vacuum tube has added impetus to work in this field. Krantz,[1] Wegel,[2] Minton,[3] Lane,[4] and others in the field of physics have made accurate determinations of the energy required for producing the minimum audible sound. Guttman[5] and Hastings and Tucker[6] have attempted the adaptation of such results to diagnosis in clinical cases. In describing their results these writers assert correctly that by the adjustment of capacity and inductance in the electric circuit the complete range of audible tones may be produced. Inspection of the results shows, however, that the measurements are usually confined to a few tones scattered over a considerable range.

The tone 'gaps' and 'islands' demonstrated by Bezold[7] are variable in both location and extent and are of considerable clinical significance. Lucae.[8] The determination of the acuity of hearing for all tones through a wide range of frequencies is made possible by means of a recently developed instrument known as the *pitch range audiometer*.

The essential features of this audiometer are a variable frequency alternating current generator connected through a series of resistances to a telephone receiver. Accessory parts are a variable speed driving motor with its controlling rheostat, and an electric tachometer which

[1] Krantz, F. W. Minimum Sound Energy for Audition. Phys. Rev., p. 384, 1921.

[2] Wegel, R. L. Physical Examination of Hearing and Aids for the Deaf. Natl. Acad. of Sci. Proc. VIII, p. 155.

[3] Minton, J. P. Physical Characteristics of Normal and Abnormal Ears. Phys. Rev., p. 80, 1922.

[4] Lane, C. E. Minimum Sound Energy for Audition for Tones of High Frequency. Phys. Rev. *19*, p. 492, 1922.

[5] Guttman, John. Further Studies in the Functional Examination of the Acuity of Hearing, etc. Laryngoscope, p. 829, 1922.

[6] Hastings, and Tucker. An Attempt to Standardize Tests of Hearing. Proc. Roy. Soc. of Medicine, *16*, Jan. 1923.

[7] Bezold. Text Book of Otology. Trans. by Holinger. p. 53.

[8] Lucae. Arch. of Otol., p. 157, 1907.

indicates the generator speed in terms of vibrations per second. In Fig. 1, *A* and *B* are two generators, the former being for tones in the upper scale and the latter for low tones. Since this part of the apparatus is by no means quiet, it is placed in a separate room at some distance from the observer. On the control board, Fig. 2, are mounted the series of resistances, *G*, which determine the amount of current which passes through the receiver; (Receiver not shown in figure.) the dial, *F*,

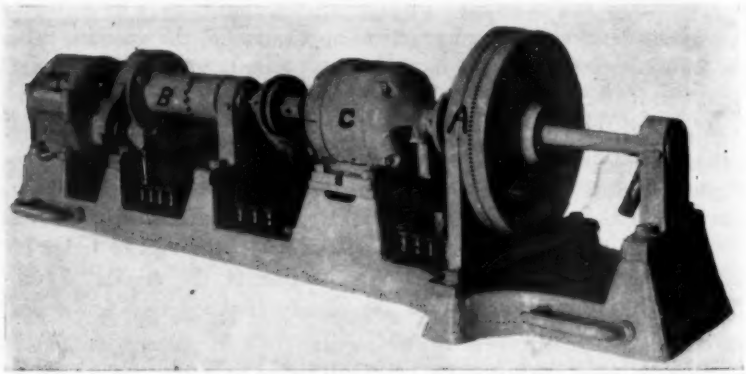


FIG. 1.

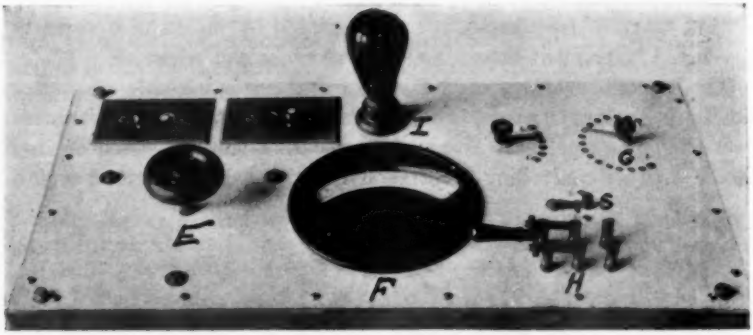


FIG. 2.

of the electric tachometer, *T;* the rheostat, *E*, for speed control; switches, *H*, which interrupt the current in the receiver; a signal lamp, *I;* and starting switches.

The motor, *C*, is a Westinghouse 1/8 H.P. compound wound D.C. motor, rewound in shunt form to give a speed of 6000 rpm., and is attached by flexible couplings to the two generators. The rheostat

is a potentiometer type specially constructed to give a range of motor speeds from 120 to 6000 rpm. *Generators.* The large wheel at *A* is of billet steel, 9 in. in diameter, which, with its shaft, rotates on ball-bearings in close proximity to an upright cast iron plate. On the periphery of the lateral surface of the steel plate are cut 150 teeth (B. & S. No. 8 cutter, 24 pitch.) in a 3/4 in. rim. Similar teeth are cut in the upright cast iron plate. Between the teeth of the two plates is an air gap of .008 in. Surrounding the shaft between the plates are two coils, a primary of 3000 turns (No. 28 enamel copper wire), and a secondary of 3000 turns (D.S.W. No. 36). Connected to the primary through a switch at *S* on the control board is a magnetizing battery of 6 volts. As the rotating wheel with its one hundred and fifty teeth turns before the teeth of the stationary wheel, the magnetic flux varies as the air gap changes. The secondary oscillating current is carried to the telephone through the resistances. The generator, *B*, is similarly constructed except that there are but 15 teeth on the lateral surface. *Resistances.* The series of resistances at *G* are wound non-inductively and vary in increasing values from .00025 ohms to approximately 1500 ohms, additions in each step being 200% of the totals. The potentiometer principle is used in making telephone connections.

Procedure. It is in the method of conducting the test that the radical difference lies between the use of this instrument and others where electricity is used as the sound source.

The examiner with the control board and the observer with the telephone receiver at his ear are located in a quiet room. The intensity is first adjusted to produce a distinctly audible tone. By some noiseless method, (electric contact key and lamp) the patient is instructed to signal as long as he hears any tone. The speed of the driving motor is then slowly accelerated and retarded until the sound in the telephone receiver is no longer audible. The highest and lowest tones audible at this intensity are then indicated on a chart similar to that shown in Fig. 3. The range of audible sounds at this intensity having been determined, the procedure is repeated at all intensities and the limits noted on the chart. These, joined by a curve, indicate the field of hearing. The procedure of slowly changing the pitch of the stimulus tone with the intensity step kept constant offers at least two distinct advantages. First, the location and extent of any defect in the tonal range is quickly and accurately determined. Second, attention is more easily controlled with a tone constantly changing in pitch as a stimulus than with a tone of constant pitch with the intensity gradually decreasing.

Results. Tonal quality. It probably cannot be said that a generator of this type will produce a pure sine wave current. It is also doubtful if a pure sine wave current would produce a tone absolutely without harmonics in a telephone receiver of the ordinary type. However,

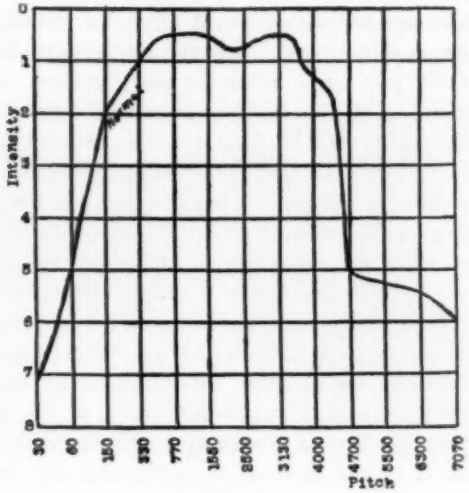


FIG. 3.

experimenters who are continually working in the field of sound have remarked at the purity of the tone, comparing it favorably with tones from tuning forks. Fig. 4 is a photograph of the sound wave produced by the generator, the frequency being 425 d.v. From these two facts

FIG. 4

and also from the fact that tone gaps very limited in extent are easily located in the examinations with the instrument, a phenomenon impossible with tones of impure quality, we are led to believe that the tone quality is quite pure, at least sufficiently so for the functional tests of hearing.

Fields of hearing. Records of examinations with the instrument are shown in Fig. 5. These two are selected from a large number of cases because of the detailed information which they reveal concerning the nature of the auditory defect.

The record at the left was secured from a patient at the University Hospital who, while swimming, forced water into the tympanic chamber of the right ear. Ear ache with acute suppuration followed, and the

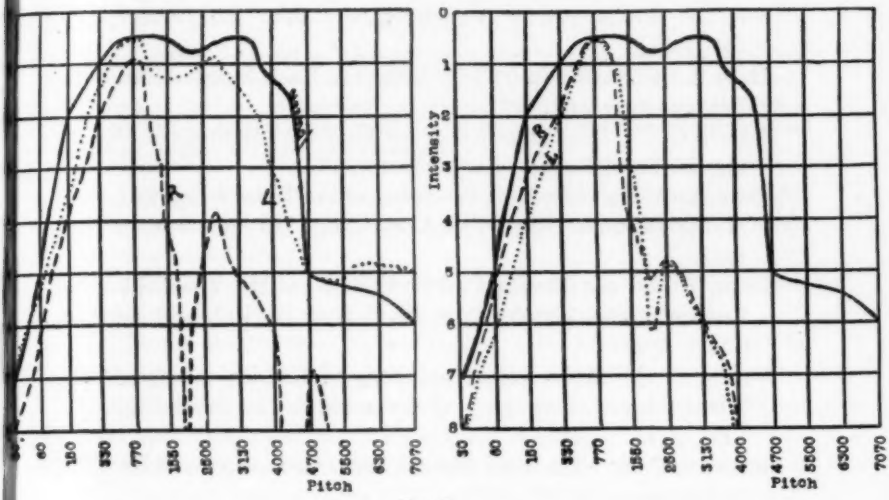


FIG. 5.

record shows the condition of the hearing thirty-six hours after the onset of the ear ache. The right ear record is indicated by the line of dashes, that for the left is in dots, while the unbroken line represents normal hearing. A complete gap of limited extent at 2000 d.v. and another between 4200 d.v. and 4700 d.v. are shown in the curve for the right ear, the left being practically normal. If we may believe Lucae,[8] these gaps indicate that the inner ear is also involved with the middle ear infection in this case.

The records at the right are the findings in a patient sixty years of age. This patient had little complaint so far as hearing was concerned. He was able to carry on conversation of ordinary life with little interference. The record shows, however, that there was a marked loss in acuity in both ears for tones above 1000 d.v., a condition often found existing in persons of advancing years.

These two illustrations are sufficient to indicate the advantages of the examination with the audiometer. Detailed reports covering an extended period of time will be found in the following articles:

Dean, L. W. and Bunch, C. C. The Use of the Pitch Range Audiometer in Otology. Laryngoscope, p. 435, 1919.

Seashore, C. E. The Iowa Pitch Range Audiometer. Journal-Lancet. p. 545, 1919.

Dean, L. W. and Bunch, C. C. Results obtained from One Year's Use of the Audiometer in the Otological Clinic. Laryngoscope, p. 137, 1921.

Dean, L. W. Studies in Otology Using the Pitch Range Audiometer. Laryngoscope, Oct. 1921.

Bunch, C. C. Demonstration of the Improved Methods of Measuring the Tonal Range. Laryngoscope, 1921.

Dean, L. W. and Bunch, C. C. Study of the Tonal Range in Lesions of the Acoustic Nerve and Labyrinth. Laryngoscope, p. 309, 1923.

Dean, L. W. and Bunch, C. C. A Study of the Tonal Range in Lesions of the Middle Ear. Ann. of Otol., Rhi. and Laryng., p. 617, 1922.

Bunch, C. C. Measurement of Acuity of Hearing throughout the Tonal Range. University of Iowa Studies in Psychology, No. VIII, p. 45.

Seashore, C. E. The Iowa Pitch Range Audiometer and Its Uses. Laryngoscope, p. 295, 1923.

UNIVERSITY OF IOWA,
IOWA CITY, IOWA.

AN AUTOMATIC FOCUSING DEVICE FOR PHOTOMICROGRAPHY WITH ULTRAVIOLET LIGHT OR LIGHT OF ANY WAVE LENGTH

By W. T. Bovie and C. E. Barr

SUMMARY

The microscope is focused on the object using a band of yellow-green light the wave lengths of which differ by only 1.63mμ. The light filter used for this purpose is described. In focusing by this light the apparatus is so adjusted that the rotation of a screw through a few turns between automatic stops brings the microscope automatically into precise focus for the light employed in making the photograph.

The special effects obtainable by the application of ultraviolet light to photomicrographic purposes have been summarized in Zeiss's Catalogue as follows:

1. The resolving power of the objectives is double that of objectives of identical numerical aperture when working with daylight.
2. Many objects showing no sign of color in white light act in relation to ultraviolet light as if they were colored objects.
3. Ultraviolet light often exerts very pronounced physiological effects upon living and surviving organic objects.

To the above must be added another effect that bids fair to rival if not to surpass any of them in ultimate usefulness to the biologist. Owing to the fact that the ultraviolet is a region of very great absorption it develops that the boundaries of objects having only slight differences in composition, that are indistinguishable by transmitted white light, become very obvious when ultraviolet light is employed not only because these small differences cause great differences in absorption but to even greater degree because of the great changes in relative refractive index which accompany these small variations. The meaning of this to the biologist is that protoplasmic structures are rendered visible that ordinarily are only revealed in killed, fixed and stained material. The differential absorption actually functions as a stain. There has always been a question as to how much of the structure shown in fixed material is natural and how much is artefact. For many purposes such as the diagnosis of morbid tissues, fixed material is quite as useful as material that contains no artefacts, provided the fixation always produces the same artefacts, but to the

biologist who is interested primarily in the living protoplasm, and who wishes to determine the significances and the fates of the structures revealed, artefacts are intolerable. To these, ultraviolet photomicrography, when properly safeguarded, offers a sure and certain assistance.

As glass is practically opaque to ultraviolet rays of wave length shorter than 290mμ, all parts of an ultraviolet photographic apparatus through which the rays are to pass, including the lenses of the microscope, must be made of a substance transparent to ultraviolet rays. Quartz is the medium usually employed. It is obvious, however, that an objective all of whose lenses are of quartz cannot be corrected for chromatic aberration and hence that it forms with white light a multitude of overlapping images of various colors and of different degrees of magnification. Owing, therefore, to the impossibility of sharply focusing on an object by white light, together with the added difficulties associated with the large difference in focus between the visual and the ultraviolet rays, it has been customary to focus in ultraviolet photography by means of an eye-piece provided with a suitable fluorescent screen. Apart from the difficulty of getting a clear picture on the fluorescent screen, the remaining and the really fatal defect of the method is that with light intensities great enough to give the needed visibility, the organism examined is necessarily radiated for so long a time during the process of focusing that the structure is materially affected. This seems to have escaped the observation of a number of workers in ultraviolet photomicrography with the result that many of the ultraviolet photographs that have been exhibited show conspicuous artefacts. The authors have examined very many such pictures and are convinced that it is only by a very rare chance that any one of them can be accepted as a true representation of the structure of the protoplasm when it was alive. The uncertainty is made all the greater when the photographer fails to give the conditions under which the exposure was made. It therefore seemed to the writers desirable and indeed necessary to devise a method by which the focusing might be accomplished by visual rays that would be innocuous to the organism while the necessary change in focal adjustment should be accomplished by a mechanical device.

This paper describes a method by which an object may be focused upon with great precision while the first ultraviolet light that strikes the object is that employed in making the negative. Several years after the device described below had been invented and used a

reference was found in the literature to another device that had been employed by Swingle and Briggs. Their method offered decided advantages over other methods in use at the time but it did not approach in precision to the method herein described and it is impossible to focus on the object with either so great accuracy or comfort.

As has been said, the lenses employed in ultraviolet photography are made of quartz and are not chromatically corrected thus giving a very confused image with white light. If, however, monochromatic light be employed extremely sharp definition may be attained. For visual focusing it has been found that a special green light is most advantageous. It is very pleasant to work with, there is no glare and the approach to true monochromatism is very close. For use with magnesium electrodes this light is filtered out by a combination of the *G* and *H* filters listed by and made by the Eastman Kodak Company. This combination transmits a band in the yellow-green extending only from wave lengths 510mμ to 540mμ. As a matter of fact, if the monochromatic illuminator of the apparatus is swung into position so that the green lines of the spectrum given by a spark between magnesium electrodes is used for focusing, all the light transmitted by this filter, that is of sufficient intensity to impress the eye, is given by lines of wave lengths 516.755, 517.287 and 518.384mμ which is monochromatic to all intents and purposes.

This filter also answers very satisfactorily when the spark passes between electrodes of cadmium, though the range in wave length is considerably wider than with those of magnesium. If, however, a moderately tinted fluorescin screen (made by staining a fixed dry-plate in fluorescin solution) is added to the *G* and *H* filters mentioned above, the somewhat divergent line at 508.6mμ practically disappears and only those at wave lengths 533.8mμ and 537.9mμ are strongly transmitted. The illumination of the field is somewhat reduced but it is ample for the most delicate focusing.

The difficulty in taking photographs with ultraviolet light is like that experienced with the earlier achromatic objectives, in that the foci for the visual and actinic rays are not coincident and a correction in focus is needed to compensate for the difference. In ultraviolet photography the correction is more difficult in practice because the distance through which the adjustment must be made is comparatively very great, demanding over 10 complete turns of the fine adjustment screw with the 6 mm objective, while owing to the great increase in numerical aperture, the adjustment must be exceedingly precise.

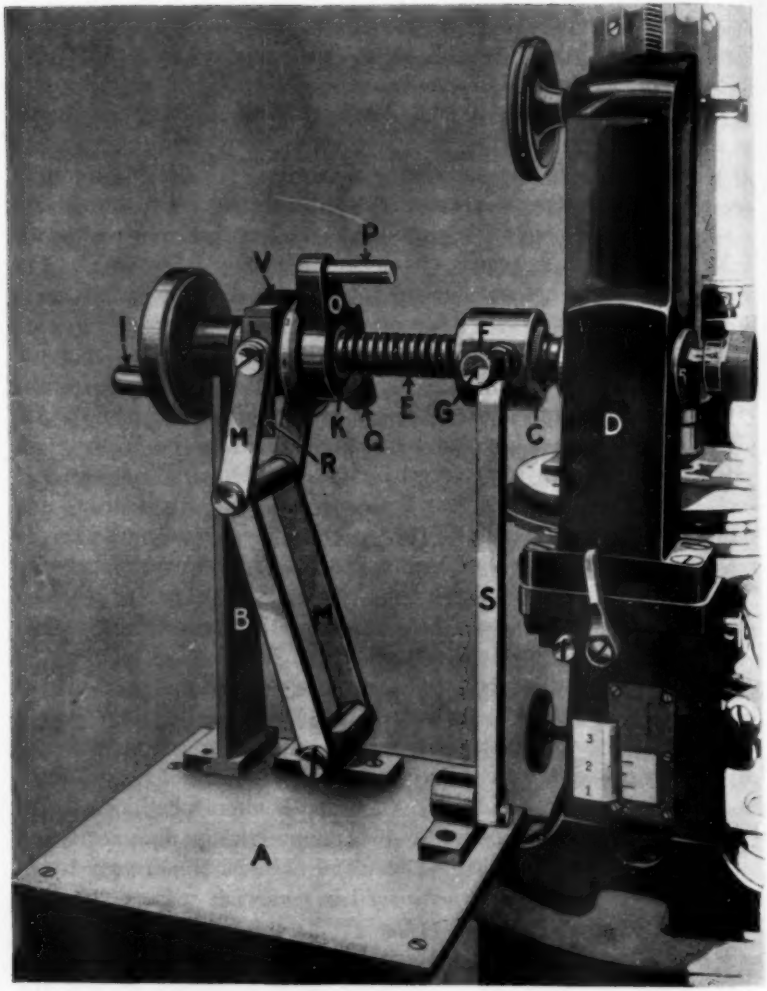


FIG. 1. *Automatic focusing device for photomicrography with ultraviolet light.*

Besides all this the ultraviolet light is invisible to the eye and the proper adjustment for focus must be found by a method of trial and error.

These difficulties are overcome by this automatic focusing device. As the object is focused on by the monochromatic light two stops are

automatically adjusted so that the simple turning of a screw between these automatic stops brings the microscope into precise focus for the ultraviolet rays.

On a base plate *A* is erected an upright *B* terminating above with a bearing the center of which is at the same height and is exactly aligned with the axis of the (side) fine adjustment *C* of the microscope *D*. Through this bearing extends the bearing of a square-threaded screw *E*, 8 threads to the inch, and 4 inches long, over all, the right end of which carries a hollow cylinder *F* that fits the head of the side fine adjustment *C*. The cylinder must be bored to fit the fine adjustment head very exactly, permitting free rotation, but no free play. It is provided with a set-screw *G* that extends outward 1 inch, which, when set, clamps the cylinder to the fine adjustment head. At the outer end of the focusing screw is a wheel *H* with handle *I* by which the screw may be turned rapidly. Engaging the focusing screw is a hollow bolt *K* threaded upon the inside to engage the threads of the focusing screw. The head *L* of this bolt is prevented from turning with the screw by the jointed frame *M* which, however, permits translation of the bolt along the focusing screw as the latter is rotated. Turning on the cylindrical shaft of the hollow bolt is a circular brass disk whose rim is divided into 100 equal parts. The disk is screwed fast to a plate *O* which carries a pin *P*. The plate may be clamped tightly on the hollow bolt by a draw-screw *Q*. On the top of the bolt-head *L* and in juxtaposition with the divided rim of the disk is a vernier *V* by which the exact angular position of the pin *P* may be ascertained for record. *R* is one of a set of brass riders bent so as to fit and hang over the screw *E* for the purpose of automatically restricting the number of free turns of the screw as described later. *S* is a post which may be laid down or which may be erected, its free end, in the latter case, forming a stop for the staff of the set-screw *G*.

The calibration of the device is simple. First, the hollow bolt *K*, with its attachments, which will hereafter be referred to as the carriage, is run toward the microscope until the staff of the set-screw *G* is stopped by the pin *P*. The screw *G* is now loosened, thereby freeing the focusing device from the microscope, and the quartz stage micrometer is carefully focused upon by ultraviolet light, using the fluorescent eyepiece. The screw *G* is then tightened. Now swing the illuminator to visual (monochromatic) light and by means of the focusing screw *E* alone (turning it by the handle *I*) run the carriage away from the

microscope until the micrometer is sharply focused by the visual light. Note the number of complete turns n of the focusing screw. Loosen the set-screw G and rotate the cylinder F toward the observer until the staff of the set-screw G rests on the post S, as shown in the Fig. 1, being careful that the fine adjustment of the microscope does not turn with it. Tighten the screw G. This leaves the pin P out of adjustment and the pin P must be turned toward the observer through approximately the same arc that the screw G has just been moved. It will be convenient at this time to prepare and set in place a brass rider R, to ride on the focusing screw, of such a width as to restrict the travel of the carriage to $n+1$ turns when the carriage is again run to the left for focusing this objective. (It will probably be found that the number of turns to be allowed for will be 2, 3 or 10, depending on the objective used.) Make notation of the vernier reading that indicates the angular position of the pin P. The change required to compensate for the difference in focal adjustment between ultraviolet and visual light has now been determined approximately. Focus by visual light, tighten G, run the carriage to the right until travel is stopped by the pin P and keeping the pin in contact with the stop, give the divided disk a series of settings on either side of the approximate setting previously determined and make trial exposures for each setting by ultraviolet light. Inspection of the resulting negatives makes it easy to select the setting for sharpest focus.

It will be observed that by means of this device it is a simple matter to ascertain the relative position of sharpest focus for any part of the spectrum with reference to the monochromatic focus adopted as the standard and automatically to make the adjustment for it with precision. The principle is quite as applicable to the infrared, for example, as to the ultraviolet.

When the number of turns of the focusing screw and the reading of the vernier for the proper position of the pin P has been once determined for any objective, ocular, and camera extension, the same settings may be employed with confidence for all future exposures with that combination so long as the relative positions of the plate A and the microscope D are unchanged.

The procedure in making an exposure when once the instrument has been calibrated for the light employed is as follows:

Hang the proper rider R for the objective used on the focusing screw at the left of the movable carriage and run the carriage to the left as far as it will go.

Set the divided disk bearing the stop *P* at the proper vernier reading.

Insert the monochromatic filter.

Swing the monochromatic illuminator so as to illuminate the object with the green part of the spectrum.

Focus on the detail desired, holding the loosened screw *G* on top of the post *S*.

Tighten the screw *G*.

Shut off the light, remove the filter and swing the monochromatic illuminator into such position as will throw a beam of light of the desired wave length into the microscope. It will be found very convenient to provide the monochromatic illuminator with a wave length scale so that it can be quickly and accurately adjusted for any wave length throughout the entire spectrum.

Swing the camera into position.

Rapidly turn the focusing screw by the handle *I* until the pin *P* is stopped by *G*.

Set the exposure shutter for the exposure desired and start the spark.

Draw the dark slide and snap the shutter.

Special attention is called to the fact that in this method the negative is made by the first ultraviolet light that falls upon the specimen. It is therefore possible to use light of very great intensity and the shutter may be set at such a speed that the exposure ends before any cytolytic changes have occurred.

We have found that living protoplasm can be exposed to brief radiation of sufficient intensity to give satisfactory negatives, without producing other than temporary stimulation, so far as we were able to detect, if the exposures are not made at too close intervals. It was not possible to do this when using light of lower intensity.

For this reason a specially intense source of light was developed, which has been described in a previous paper. Further, it is only by using a light of high intensity that one may employ high-power oculars without destroying the organization of the protoplasm, while the high initial magnification given by such oculars is of great service in minifying the indistinctness caused by the grain of the plate.

Several thousand photographs have been made with the aid of this device and it may be confidently claimed that in none of them are the results vitiated by cytolytic changes in the structure or organization of the protoplasm. Not all the photographs are successful, for

even in the short period of time that elapses between the focusing and the exposure a living organism often may so move that the particular part focused on is carried out of the plane of distinct vision. When the object is quiet the focus is precise.

BIOPHYSICS LABORATORY,
HARVARD UNIVERSITY,
CAMBRIDGE, MASS.

Determination of the Mass of the Particle Emitting the Many-lined Spectrum of Hydrogen.—The breadths of thirty-five lines in the orange and blue regions of the many-lined spectrum, emitted by hydrogen in a water cooled discharge tube, was estimated by Michelson's method of observing the visibility of interference fringes of higher orders. All the lines are found to be unmistakably narrower than they would be if they were emitted by particles of the mass of the hydrogen atom, in thermal agitation at the temperature of the water. The conclusion, in agreement with that of Merton based on direct measurement of photographed lines, is that the many-lined spectrum is emitted by H_2 molecules. An earlier discordant result by Buisson and Fabry is now conceded by them to have been in error. [Duffieux, C. R. *176*, pp. 1876-1879; 1923.]

K. K. DARROW

THE COLORIMETRY AND PHOTOMETRY OF DAYLIGHT AND INCANDESCENT ILLUMINANTS BY THE METHOD OF ROTATORY DISPERSION*

By Irwin G. Priest

SYNOPSIS

Further studies, both theoretical and experimental, have been made on the method of photometry and colorimetry previously proposed by the author.

In the light of this work the method is now proposed as a complete and satisfactory solution of the practical problem of the visual photometry and colorimetry of all illuminants (including the important phases of daylight) whose spectral distribution approximates the Planckian formula closely enough to give a color match. This solution is based upon the principle of the additivity of homogeneous luminosities and the assumption of a standard visibility function.

The method falls in the general class of substitution "equality of brightness" methods. All brightness matches are made at a color match. This color match is obtained by modifying the color of a constant comparison source by allowing its light to pass through a train of nicol prisms and quartz plates which form, in effect, a blue or yellow filter of continuously adjustable spectral transmission.

Tables and graphs have been prepared by which color temperature and intensity may be readily obtained from the instrument readings on the basis of any visibility which it is desired to assume as standard.

CONTENTS

		Page
I	Introduction	1176
	1. General	1176
	2. Nomenclature	1176
II	The grading of illuminants with reference to quality of color	1179
	1. The temperature of the source as an index of the quality of color	1179
	2. The principle of the rotatory dispersion filter	1183
	3. Calibration of the filter in terms of color temperature	1188
	4. Experimental procedure for the measurement of color temperature by the rotatory dispersion method	1189
	a General procedure	1189
	b Special cases	1190
	5. The spectral centroid of light as an index of the quality of color	1190

* Amplification and extension of a paper presented at the Seventh Annual Meeting of the Optical Society of America, Washington, October 28, 1922. (J. O. S. A. and R. S. I., 7, pp. 75-76; 1923.) Published by permission of the Director of the Bureau of Standards and the Director of the Munsell Research Laboratory. The work on which this paper is based was conducted in part while the author was an employee of the Bureau of Standards and in part while he was Research Associate of the Munsell Color Company, stationed at the Bureau of Standards. A more detailed paper will appear later under the same title as a *Scientific Paper of the Bureau of Standards*. The present paper is, however, an adequate presentation. The extensions in the Bureau paper will include merely more *detailed* tables and more supporting evidence.

The author is indebted to Mr. C. L. Snow for his careful preparation of the figures.

III The measurement of intensity 1192
1. The problem of the photometry of illuminants having different qualities of color 1192
a The essential difficulties 1192
b The general practical solution in terms of standard values of visibility and the theorem of the additivity of homogeneous lights 1193
2. The measurement of relative intensity by the rotatory dispersion photometer 1195
3. Discussion of visibility data and the choice of standard values of visibility 1197
IV The instrument and its use 1199
V Conclusion 1203
Appendix: Tables 1205

I. INTRODUCTION

1. GENERAL

The methods of colorimetry and photometry to be presented in this paper have been described in a preliminary, incomplete and somewhat fragmentary manner in several previous papers.[1]

The purpose of the present communication, which contains considerable new material not published heretofore, is to present the subject in a more unified form, accompanied by a description of the "Colorimetric Photometer" which has been constructed to exemplify these methods.[2] It may be considered as definitely superseding all of these previous papers except B. S. Sci. Pap. 443, "Measurement of the Color Temperature of the More Efficient Artificial Light Sources," which contains some data which will not be fully reproduced here. Acquaintance with these previous papers need not be assumed as a prerequisite for the understanding of the subject as it is to be presented now.

2. NOMENCLATURE[3]

COLOR is the general name for all sensations[4] which are referred to the visual sense.

[1] Priest. Phys. Rev. (2) *6*, p. 64; 1915. Phys. Rev. (2) *9*, p. 341; 1917. Phys. Rev. (2) *10*, p. 208; 1917. J. Op. Soc. Am. *5*, p. 178; 1921. B. S. Sci. Pap. No. 443; July, 1922. J. Op. Soc. Am. *7*, p. 75, 1923.

[2] This photometer was exhibited at the Washington meeting of the Optical Society when this paper was presented, Oct. 28, 1922. (J. O. S. A. and R. S. I. *7*, p. 75-76.)

[3] To save space and needless duplication, explicit definitions are given only for terms which might not be understood generally by photometricians in the sense in which they are here intended by the author. All technical photometric terms used but not explicitly defined in the paper are to be understood in the sense of the definitions given by the I. E. S. Committee on Nomenclature and Standards.

[4] Sensation is here understood to mean a discrete element of consciousness, normally dependent upon the normal functioning of a sense organ. Colors are thus the discrete

In general, three terms are necessary and sufficient to completely describe any color. These three terms are: BRILLIANCE, HUE, and SATURATION. They are commonly said to designate the three *attributes* of color.

BRILLIANCE[5] is that attribute of color without which it cannot exist. All colors possess brilliance, and fall naturally into two classes, namely, (1) those which possess no other attribute, (2) those which possess other attributes in addition to brilliance. Those which possess no other attribute than brilliance are commonly called *grays*. With respect to brilliance alone any color may be rated as equivalent to some member of a series of grays of which black and white are the terminal members.

HUE is that attribute of some colors by virtue of which they are not gray, and in respect of which they fall into classes which may be designated as reddish, yellowish, greenish or bluish.

SATURATION expresses the degree to which a color exhibits hue or (from the opposite point of view) the degree to which it departs from gray. Colors differing only slightly from gray are said to be of low saturation, while those approaching a maximum departure from gray are said to be of high saturation.[6]

In the present paper, we shall find it convenient to designate as QUALITY that aspect of color which comprehends both hue and saturation.

The concepts of color and its three attributes may be formulated entirely by introspective analysis without any reference to or knowledge of its stimulus. They are purely psychological concepts. We have now, however, to consider the normal relations subsisting between color and its usual and practical stimulus from the point of view of the illuminating engineer. This usual stimulus is *radiant energy*. Strictly,

---

elements of which the visual consciousness is composed. All that is immediately contributed to consciousness by monocular vision is a pattern of colors composing the field of view. The visual recognition of shapes and forms is possible only as a result of sensibility to color differences. Therefore, form cannot logically be placed in the same category with color as is sometimes done; it is not elemental in the same sense.

[5] Luminosity and "brightness" have been generally used in this sense. Their use as sensation terms is objectionable because of their other better recognized connotations which are distinctly physical, referring to the *stimulus*.

[6] All of the foregoing definitions, while not *verbatim* copies, are entirely consistent with the definitions given in the "Report of the Colorimetry Committee," Jour. Opt. Soc. Am., August, 1922. Reference should be made to this report for a more detailed discussion. They are not entirely consistent with the definitions given by the I. E. S. Committee on Nomenclature and Standards (e.g. definition 18, in the Report for 1918). A full statement of the arguments *pro* and *con* will be found in footnote 2, p. 531, Jour. Opt. Soc. Am., August, 1922.

the color is determined not solely by the radiant energy but also, in part, by the circumstances or conditions under which the radiant energy is incident upon the retina. These conditions include: (1) the type of the individual observer's visual system, (2) the particular part of the retina receiving the energy, (3) the size of the field, (4) the state of adaptation of the visual apparatus resulting from previous as well as present stimulation, (5) the simultaneous stimulation of other parts of the retina.[7]

The necessity of a consideration of these circumstances in the present discussion is almost eliminated by the technic of the method which maintains these conditions substantially constant. We are to be concerned with an experimental method which involves only the *matching* of colors in a photometric field, a null difference substitution method.

For our present purpose then, the relations between stimulus (radiant energy) and sensation (color) may be stated as follows:

Light is radiant energy multiplied by visibility.

Color is the visual sensation evoked by the incidence of radiant energy upon the retina.

Brilliance is determined by:—

(1) the time rate at which radiant energy is incident per unit area upon the retina; and

(2) the visibility of this radiant energy.

Or, more briefly:—

Brilliance is determined by luminous flux per unit area of the retina. [The physical analogue of brilliance is brightness. In terms of Fechner's Law (which, however, is not universally valid) brilliance might even be *defined* as the logarithm of brightness.]

Quality is determined by the relative spectral distribution of the radiant energy.

If the stimulus contains much more energy of long wave-length than of short wave-length, the quality of the color will, in general, be yellowish or reddish. If the converse is true the quality will, in general, be bluish. For some spectral distribution, not at present very precisely defined, but approximately that of average noon sunlight, the color will be gray. The spectral distribution constitutes a specification of the quality of color in the sense that stimuli of the same spectral distribution must evoke the same color; but it must also be remembered that

[7] Cf. "Report of the Colorimetry Committee . . . " Jour. Opt. Soc. Am., *6*, p. 538; 1922.

the same color may be evoked by stimuli of widely different spectral distributions.

In the present paper we are concerned primarily with the following problems relating to incandescent sources:

(1) The evaluation of radiant energy, with respect to the quality of color which it will evoke.

(2) The measurement of relative luminous flux.

II. THE GRADING OF ILLUMINANTS WITH REFERENCE TO QUALITY OF COLOR

1. THE TEMPERATURE OF THE SOURCE AS AN INDEX OF THE QUALITY OF COLOR.

By far the largest and, from a practical point of view, the most important class of illuminants consists of incandescent solids. From the earliest times, probably before historic record begins, the intimate relation existing between the temperature of an incandescent solid and the quality of color must have been vaguely recognized. Quite likely the first estimates of high temperatures, even antedating by many centuries any definite concept of temperature, were made in terms of color by some primeval smith at his forge. In the course of centuries this vague idea gradually crystallized into more definite concepts until, within the last two decades, it has attained the dignity of precise scientific formulation.

At temperatures not much above that at which they become incandescent, solids are red; as the temperature increases, they become orange, then yellow, and, passing through paler and paler tints of yellow, approximate to white. The limits of terrestrial temperatures do not permit of observations beyond this stage; but we observe that, relative to the sun, our hottest artificial sources (even the crater of the carbon arc) are yellow, while many of the fixed stars are decidedly blue. The change in the stimulus giving rise to this change in sensation may be qualitatively described thus: As the temperature increases the ratio of energy of short wave-lengths to the energy of long wave-lengths also increases.

This somewhat vague and indefinite statement may now be put into precise terms and indeed into a mathematical formula, which is necessary for our purpose. Numerous investigations have shown that, to an approximation sufficient in the present problem, the spectral distribution of energy of our common incandescent sources may be expressed as a function of temperature by Planck's distribution formula, which,

in the first instance, applies strictly only to a hypothetical source now commonly known as a "complete radiator" or "black body" and which, to be specific, may well be called a "Planckian radiator," as indicating a radiator which does conform to this law. This formula is

$$E_\lambda = c_1 \lambda^{-5} (e^{\frac{c_2}{\lambda T}} - 1)^{-1} \tag{1}$$

where

E_λ is the energy per unit wave-length at wave-length λ;

T is the absolute temperature;

e is the base of natural logarithms;

c_1 and c_2 are constants.

The following remarks apply to this formula and the quantities involved throughout this paper:—

We are not concerned with the absolute value of c_1.

We are concerned only with relative values of E_λ as a function of λ for any desired T and not with relative values of E_λ as a function of T with λ constant.

T is expressed in degrees K (absolute temperature).

$c_2 = 14350$ micron-degrees.

Typical forms of this function for different values of T are shown in Fig. 1.

The color temperature of a source is the temperature at which a Planckian radiator would emit radiant energy competent to evoke a color of the same quality as that evoked by the radiant energy from the source in question.

The color temperature is not necessarily the same as the "true temperature" of the source; but this circumstance has no significance whatever in the use of the color temperature as a means to the end of establishing a scale for the quality of the color of illuminants. For this purpose no knowledge of the temperature of the source nor indeed of its emissive properties is required. *All that is involved in giving the color temperature of any illuminant is the affirmation that the color of the illuminant is of the same quality as the color of a Planckian radiator at the given temperature.*

As we shall see later, we may specify on the color temperature scale, even colors which owe the constitution of their stimuli to quite different causes than merely the temperature of a source, as for example to selective scattering or selective transmission. Thus we may measure the color temperature of the blue sky or the overcast sky. In giving such a color specification we merely affirm that the sky has a color of the same quality as a hypothetical Planckian radiator at the specified temperature. Likewise, we may specify the color temperature of an

artificial daylight unit consisting of an incandescent lamp and a blue filter (such as have been introduced by H. E. Ives, M. Luckiesh and others). In this case there is no actual source at any temperature at all comparable with the color temperature found; but the radiant energy emergent from the filter is competent to evoke a color of the same quality as the color of a Planckian radiator at the given temperature.[8]

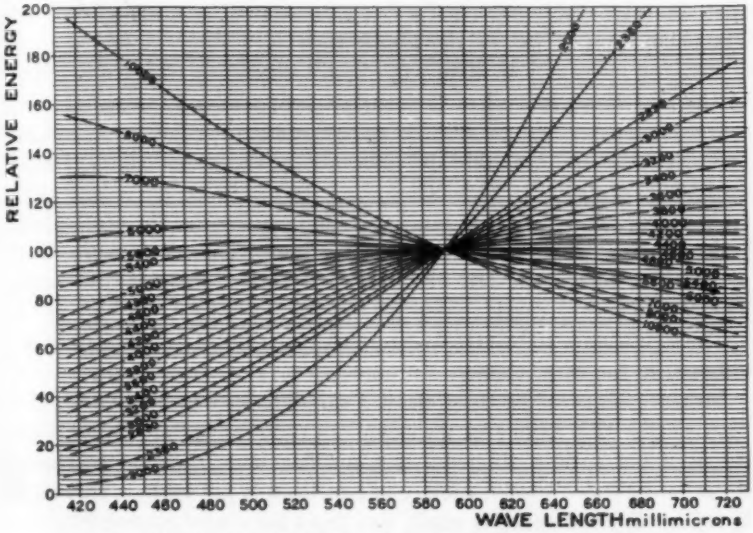


FIG. 1. *Planckian distributions of energy at various temperatures.*

Planck's constant, $c_2 = 14350$ micron-degrees. Numbers attached to the curves indicate temperatures in degrees K (absolute temperature). Ordinates of the different curves are arbitrarily made equal at wave-length 590.

To recapitulate, the only requirement that need be satisfied by a color stimulus in order that it may be specified in terms of color temperature is that it be competent to evoke a color of the same quality as the color which would be evoked by a Planckian radiator at some temperature. In connection with this last statement, however, it is to be noted that color stimuli amenable to specification in terms of color temperature fall into three general classes, namely:—

[8] In such extreme cases, where the temperature is such a purely hypothetical thing, I have heretofore sometimes spoken of "apparent color temperature" but this expression is essentially redundant, since *all* color temperatures are merely apparent temperatures. H. E. Ives has suggested that, in case our primary concern is the specification of color rather than temperature, we might well speak of "temperature color."

Class 1.—Those for which the spectral distribution of energy is identical with that given by the Planckian formula.

Class 2.—Those for which the spectral distribution of energy is not identical with that given by the Planckian formula, but still is of such a form that the quality of the color evoked is the same as would be evoked by the energy from a Planckian radiator at the given color temperature.

Class 3.—Those for which the spectral distribution of energy is such that the color can be matched *only approximately* by a stimulus of the Planckian form of spectral distribution.

Perhaps no practical illuminant fulfills rigorously the requirement of the first class; but all of the incandescent filament lamps (carbon, vacuum tungsten and gas-filled tungsten) approximate it very closely.[9] Overcast skies and blue skies fall generally in the second class. Direct sunlight near noon for high atmospheric transmission falls also in the second class. When the sun is near the horizon or when the atmosphere is dust-laden, it falls in the third class. Some artificial daylight units of the blue filter type fall in the second class, others in the third. The Moore carbon dioxide lamp falls in the third class.

The significance to be attached to the experimental data is somewhat different in the three classes.

Measurements in the third class, if the difference in quality of color between the test light and the complete radiator is much more than the least perceptible difference, are nothing more than poor "make-shifts" for a color specification. The method is not really applicable. If the quality difference is small, one finds the *closest approach* to the color on the color temperature scale and notes that there is still a small difference which may be described *qualitatively*. However, these cases seldom occur in the case of common practical illuminants and the important phases of daylight.

In the second class, observations are made with as great ease as in the first, but the data obtained have not strictly the same significance. In particular: (1) the results may be affected by a personal equation due to the particular type of the observer's color vision; (2) the illuminant tested *may* not produce the same effect in the illumination of selectively absorbing material as would be produced by a Planckian radiator at the color temperature found. These considerations which apply particularly to the artificial daylight units are probably very

[9] Cf. B. S. Sci. Pap. No. 443, Fig. 1.

seldom of any practical importance; and insofar as they may be of consequence, they represent shortcomings of the artificial daylight rather than defects of the method of measurement.

In the first class we have to do with more definite conditions and may speak with less qualification. If we know in advance that the illuminant is characterized by a Planckian spectral distribution of energy,[10] then a determination of its color temperature is tantamount to an experimental determination of its actual spectral distribution, for this latter may be obtained by substituting the observed value of color temperature (T) in the Planckian formula.

The concept of color temperature and its application to the grading of illuminants has been developed most notably by Hyde and his associates and also to a less extent by other previous investigators. These workers have measured color temperatures up to about 3200°K. Even this is higher than it is feasible to actually operate a complete radiator as a standard; and for higher temperatures it has been necessary to have recourse to an entirely different technic than that used by previous workers. We have next to describe this technic but, before proceeding, the reader may do well to fix his ideas of the color temperature scale in terms of actual illuminants by referring to the right-hand scale in Fig. 2.

2. THE PRINCIPLE OF THE ROTATORY DISPERSION FILTER.

If it were possible to operate a Planckian radiator at any desired temperature the obvious and immediate procedure for the determination of any color temperature would be:—

(1) Illuminate the two halves of a photometric field respectively by the illuminant in question and the Planckian radiator.

(2) Adjust the temperature of the latter to match the field as to quality of color, a brilliance match being simultaneously attained by some suitable auxiliary adjustment. This temperature would be, by definition, the desired color temperature.

On account of the experimental difficulties of operating the Planckian radiator and measuring its temperature, such a procedure would be very inconvenient at any temperature, and, since it would be impracticable at temperatures above about 2000° K and utterly impossible above 3800° K, the primary desideratum for color temperature measurements is some substitute procedure by which radiant energy of Planckian distribution corresponding to a conveniently adjustable value of T can be delivered to the observer's eye *via* one side of a

[10] That is, a distribution which may be represented by the Planckian formula for *some* value of T *which need not be the true temperature.*

photometric field. We might imagine an hypothetical device somewhat as follows serving this purpose. It might consist, for example, of:—

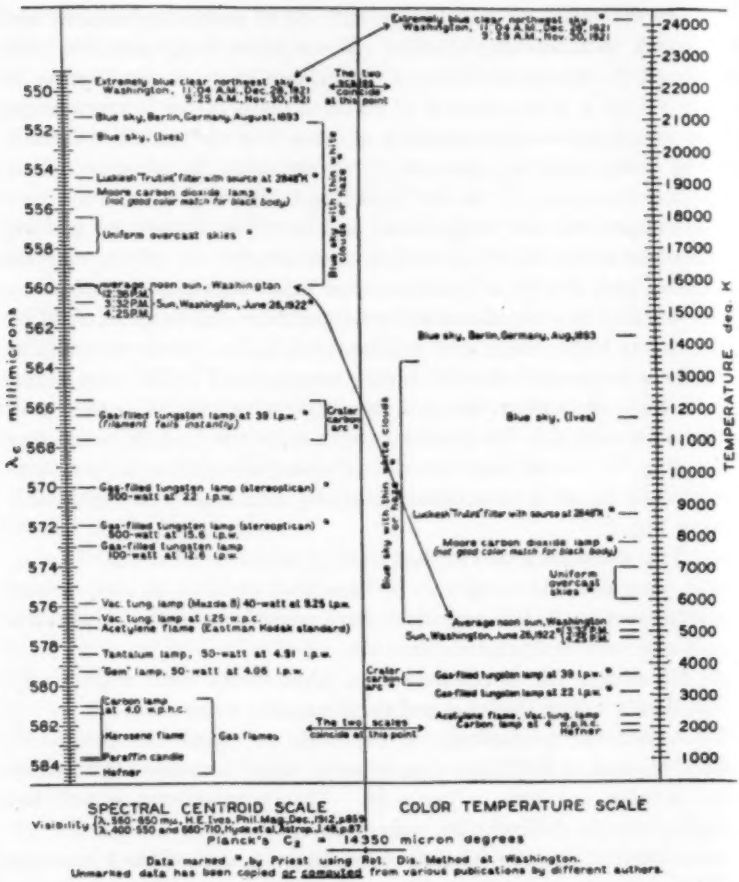

SPECTRAL CENTROID SCALE
COLOR TEMPERATURE SCALE
λc millimicrons
TEMPERATURE deg K
Planck's C2 = 14350 micron degrees
Data marked *, by Priest using Rat. Dis. Method at Washington.
Unmarked data has been copied or compiled from various publications by different authors.

FIG. 2. *Daylight and artificial illuminants on the scale of color temperature and on the scale of the spectral centroid of light.*

Comparison of these two scales shows how the yellower sources are relatively crowded together and the bluer sources relatively dispersed on the color temperature scale.

The spectral centroid scale is based on values of visibility given in column 2, Table 5.

(1) A standard incandescent lamp emitting radiant energy of known spectral distribution; and

(2) Between the lamp and the observer's eye, some sort of radiation filtering device of adjustable spectral transmission under the control of the observer, but subject to some automatic constraint such that, for all adjustments, the spectral distribution of energy emergent from the filter must conform to the Planckian distribution formula. That is, the mechanical adjustment made by the observer would correspond merely to varying T in the Planckian formula.

This is precisely the type of device which is practically realized in the Rotatory Dispersion Colorimetric Photometer, now to be described.

The essentially characteristic parts of this instrument are shown in Fig. 3.

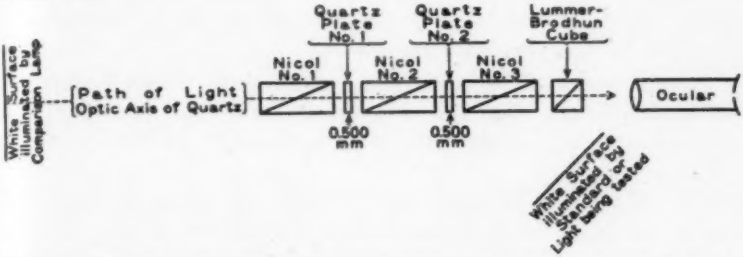


FIG. 3. *Essentially characteristic parts of the rotatory dispersion colorimetric photometer.*

The rotatory dispersion filter is used in two forms which may be conveniently designated as the simple type and the duplex type. The duplex type consists of the following parts named in order from the source to the eye:—

Nicol prism No. 1
Quartz plate No. 1
Nicol prism No. 2
Quartz plate No. 2
Nicol prism No. 3
} (See Fig. 3)

The simple type consists of only two nicol prisms with one quartz plate between them, i.e., it may be considered as derived from the above duplex type by eliminating nicol No. 1 and quartz plate No. 1. These filtering devices produce the same net effect (absorption of radiant energy selective with respect to wave-length) as that produced by stained glass or dyed gelatine, although their constitution and mode of operation are quite different. In their application and usefulness to our problem, they differ from these latter in that their spectral transmission is continuously and conveniently variable, being a function

of the angle between corresponding planes through the nicols as shown by the formulas (2 and 3) below.

It may be said at once that, in the method as now developed, the simple type suffices for all temperatures up to about 4200° K, i.e., for all of the artificial incandescent illuminants, while it is necessary to use the duplex type for temperatures higher than this, i.e., for daylight.

Considering first the simple type, we have the following fundamental formula for the relative transmission as a function of wave-length.[11]

$$\sin^2(\phi - b a_\lambda) \tag{2}$$

where

a_λ is the rotation of the plane of polarization by the quartz per mm of the path in the plate, for radiant energy of wave-length λ;

b is the thickness of the plate in mm;

ϕ, measured from the extinction position with quartz removed, is the angle through which the nicol nearest the eye has been rotated in the same direction as the rotation of the plane of polarization by the quartz.

The value of a_λ is known and tabulated in the literature for all values of λ in the visible spectrum with much greater accuracy than is needed for our purpose.

The value of b may be obtained to the required accuracy, (about 0.001 mm) by a mechanical micrometer measurement of the thickness of the quartz plate.

Knowing a_λ and b, we can readily compute, for any desired value of ϕ, a series of values of $\sin^2(\phi - b a_\lambda)$ corresponding to a series of values of λ. The plot of these values of $\sin^2(\phi - b a_\lambda)$ against λ will show the relative spectral transmission of the filter for the assumed value of ϕ.

Likewise, the relative spectral transmission of the duplex filter may be found by the formula

$$\sin^2(\phi_1 - b_1 a_\lambda)\sin^2(\phi_2 - b_2 a_\lambda) \tag{3}$$

where (See Fig. 3)

[11] It is not possible to take space to even outline here the elementary knowledge of the polarization of light, the rotation of the plane of polarization by crystalline quartz and the rotatory dispersion associated with this rotation. This information may be found in the general text books on physical optics; and, on the basis of it, the formulas here given for the relative transmission of the system of nicol prisms and quartz plates may be readily derived. The reader who does not care to go into this may take it as an *empiric fact* that a quartz plate placed between two nicol prisms so that the path of light (line of sight) is coincident with the crystalline optic axis does constitute a selective filter of radiant energy; and he must take, on faith, the formulas (2 and 3) which give the relative transmission of such filters as a function of wave-length.

b_1 is the thickness in mm of quartz plate No. 1;

b_2 is the thickness in mm of quartz plate No. 2;

ϕ_1 is the angle through which nicol No. 1 has been rotated from the crossed position relative to nicol No. 2, in a direction *contrary* to the direction of rotation of the plane of polarization in quartz plate No. 1;

ϕ_2 is the angle through which nicol No. 3 has been rotated from the crossed position relative to nicol No. 2 in the *same direction* as the rotation of the plane of polarization in quartz plate No. 2.

If $(E_\lambda)_s$ represents the radiant energy of the lamp as a function of wave-length, the relative value of energy (as a function of wave-length) emergent from the filter will be given for the simple filter by

$$E_\lambda = (E_\lambda)_s \sin^2(\phi - b a_\lambda) \tag{4}$$

and, for the duplex filter, by

$$E_\lambda = (E_\lambda)_s \sin^2(\phi_1 - b_1 a_\lambda) \sin^2(\phi_2 - b_2 a_\lambda). \tag{5}$$

A plot of either of these functions of λ for a lamp of given spectral distribution and given values of ϕ (or ϕ_1 and ϕ_2) will show the relative spectral distribution of energy emergent from the filter for the given lamp and the given value of ϕ (or ϕ_1 and ϕ_2).

The significance and importance of these formulas for our purpose lie in the following relations.

If $b = b_1 = b_2 = 0.500$ mm and $\phi_1 = 170°$, then it has been found that, for certain standard sources of known energy distribution, $(E_\lambda)_s$, these continuously varying functions of ϕ or ϕ_2 (the spectral distributions of energy given by formulas 4 and 5) may be duplicated to a close approximation[12] *by the Planckian function of T over a considerable range of values of ϕ or ϕ_2.*

To state this more graphically, let ϕ', ϕ'', ϕ''' be particular values of ϕ. Then if $(E_\lambda)_s \sin^2(\phi' - 0.5 a_\lambda)$ be plotted against λ, we obtain a curve which can be duplicated to a close approximation by the Planckian function plotted against λ for some value of T, say T'. Likewise for ϕ'', ϕ''', etc., other values of T, say T'', T''' etc., will be found in the same relation. It is apparent then that, for the given standard source and the range of values of ϕ over which this relation holds, we may plot a curve[13] showing T as a function of ϕ.

[12] The accuracy with which Planckian spectral distribution curves can be duplicated by the use of the rotatory dispersion filter, has been illustrated heretofore (B. S. Sci. Pap. No. 417, Fig. 7, p. 242, and B. S. Sci. Pap., No. 443, Fig. 6, p. 230); and will be illustrated and discussed more fully in the B. S. Sci. Pap. edition of the present paper.

[13] The technic of constructing such curves will be explained more fully below.

In a similar manner T may be shown as a function of ϕ_2 in the case of the duplex filter used with a suitable standard source, ϕ_1 being a constant $=170°$.

In all that follows, the constants

$$b_1 = b_2 = 0.500 \text{ mm}$$

and

$\phi_1 = 170°$ when quartz plate No. 1 (Fig. 3) is in the path of light are assumed.

It is obvious from the above relations that the value of T corresponding to any value of ϕ or ϕ_2 is the value which applied in the Planckian formula will give the spectral distribution of the energy emergent from the filter for that value of ϕ or ϕ_2. In other words it is the color temperature of a source which would radiate energy of the same spectral distribution as the energy emergent from the filter.

In their practical application, these curves showing T as a function of ϕ or ϕ_2 are used exactly like any "calibration curve" where ϕ (or ϕ_2) is the "instrument reading" and T is the quantity required. It is to be observed however that they are not *empiric* calibration curves, but have been derived solely by computation from the constants of the apparatus and the standard source.

3. Calibration of the Filter in Terms of Color Temperature

In order to avoid undue complexity in the preceding section, detailed description of the procedure followed in calibrating the rotatory dispersion filter in terms of color temperature (i.e. of constructing the plot of T as a function of ϕ or ϕ_2 for a standard source) was omitted. We now return to this.

This relation between ϕ (or ϕ_2) and T may be established by several different methods of computation. That given below is by far the most reliable, and other methods which have been used tentatively heretofore will not be discussed.

The spectral centroid of any sample of light is defined as[14]

$$\lambda_c \equiv \frac{\int_0^\infty V_\lambda E_\lambda \lambda d\lambda}{\int_0^\infty V_\lambda E_\lambda d\lambda} \tag{6}$$

where V_λ is relative visibility;
E_λ is energy per unit wave-length;
both being functions of the wave-length, λ.

[14] In graphic terms λ_c is the λ-coordinate of the "center of gravity" of the area included between the "luminosity curve" and the λ-axis.

In all the computations on which our calibrations are based λ_c has been actually computed by the formula

$$\lambda_c = \frac{\Sigma V_\lambda E_\lambda \lambda}{\Sigma V_\lambda E_\lambda}$$

values of V_λ, E_λ and λ being taken at intervals of 10 millimicrons, from about 410 to 720 millimicrons.

The steps in the computation leading to the expression of T as a function of ϕ (or ϕ_2) are as follows:—

(1) λ_c is computed for a series of values of ϕ (or ϕ_2 as the case may be), the values of E_λ to be used in formula 7 being obtained from formula 4 (or 5).

λ_c is then plotted as a function of ϕ or ϕ_2.

(2) λ_c is computed for a series of values of T the values of E_λ to be used in formula 7 being obtained by the Planckian formula [1].

λ_c is then plotted as a function of T.

(3) Values of T and ϕ (or ϕ_2) characterized by a common value of λ_c are then taken as corresponding values; and T is plotted as a function of ϕ (or ϕ_2).

In all of this work the values of V_λ are those previously taken by the author as fairly representative of all available data. They are tabulated in Col. 2, Tab. 5 and shown graphically by the continuous line in Fig. 4.

It is to be noted, however, that since E_λ is of so nearly the same form in the two cases and since the same values of V_λ are used in step 2 as in step 1 just above, the values of T found by this method will be independent of the particular values of V_λ which are used.

4. EXPERIMENTAL PROCEDURE FOR THE MEASUREMENT OF COLOR TEMPERATURE BY THE ROTATORY DISPERSION METHOD.

(See Fig. 3. In certain special cases, to be noted below, nicol No. 1 and quartz plate No. 1 are not used and may be considered as removed from the figure. In one case they will be used.)

a) General Procedure.

(1) The white photometer screens so marked are illuminated respectively by the standard lamp and the comparison lamp.

(2) Both quartz plates being removed from the path of light, the current in the comparison lamp is adjusted so that the photometric field is matched for quality of color, a brilliance match being simul-

taneously made by some other suitable adjustment. This current in the comparison lamp is thenceforth maintained constant.

(3) The standard lamp is now replaced by the test lamp.

(4) The quartz plate (or plates as the case may require) being replaced in the path of light (as shown in Fig. 3), nicol No. 3 is rotated to match the field for quality of color, a brilliance match being simultaneously made by some other suitable adjustment.

(5) The angle (known as ϕ or ϕ_2, according as one or two quartz plates have been used) is read from the divided circle attached to this nicol (No. 3) and T is obtained from the calibration described above.

b) *Special Cases.*

(1) *For color temperatures between 1800° and 2700° K*, the standard lamp is a carbon filament at a color temperature of 2077° K ("4 watt-per-candle carbon," the same type as has been used for years to maintain the fundamental standard of intensity), used with the simple filter (nicols 2 and 3 and quartz plate 2).

(2) *For color temperatures between 2100° and 3200° K*, the standard lamp is a vacuum tungsten filament at a color temperature of 2360°K (the quality of color of the standard E. K. acetylene flame), used with the simple filter.

(3) *For color temperatures between 3100° and 4200° K*, the standard lamp is a gas-filled tungsten at a color temperature of 2848° K (See B. S. Sci. Pap. 443), used with the simple filter.

(4) *For color temperatures between 4200° and 24,000° K*, the standard lamp is a gas-filled tungsten at a color temperature of 2848° K (same as case 3), used with the duplex filter (nicols 1, 2, and 3 and quartz plates 1 and 2 as shown in Fig. 3).

5. The Spectral Centroid of Light as an Index of the Quality of Color.

In the determination of color temperature as outlined above it will be recalled that the temperature is obtained by way of the spectral centroid of light as an intermediate step in the computation. This suggests that the spectral centroid itself (λ_c) might be used as an index of the quality of color. We have now to inquire as to what merit there may be in this suggestion. The principal consideration to recommend this suggestion is the fact that the observational precision of measurement of λ_c appears to be a constant independent of the color temperature, whereas the observational precision with regard to temperature itself falls off very rapidly with increasing temperature.

To state this somewhat more concretely, the probable error of λ_c as computed from the observed residuals of ϕ (or ϕ_2) in any set of observations, is constant, i.e. independent of the particular value of λ_c (or T) being measured. This statement is based on a very large amount of observational data which cannot be reproduced here; but these data warrant the statement with the following qualifications:

(1) That the range of color temperature actually covered by the experimental data lies between about 3000° K and 20,000° K. (The author is of the opinion that substantially the same probable error would be found for temperatures between 2000° and 3000°.)

(2) By constancy in this case is meant that the probable error of a single observation lies in general between 0.1 and 0.2 millimicrons[15] without showing any indication of systematic variation with temperature.

Now, the significance of this psycho-physical relation from the point of view of adopting a *scale* for the quality of color of illuminants is this:—

The least perceptible difference in quality of color will correspond to a scale interval which is the same at all points of the spectral centroid scale, but subject to enormous variation on the temperature scale. For example, a just perceptible difference in quality of color at low temperatures will correspond to a temperature difference of from 1° to 10° whereas at extremely high temperatures it will be 100° or 1000°. In other words, at high temperatures, color quality changes much more slowly with temperature than at low temperatures. This results in a very distorted and misleading sort of scale. For example, a difference of 100° in color temperature at 2000° would indicate a notable difference in color; at 10,000° and above, it would be negligible. On a temperature scale of equal parts all of the yellower sources are crowded into a small region and the blue ones spread out sparsely over a long stretch, (right hand scale, Fig. 2) while on a spectral centroid scale they are more uniformly distributed (left hand scale, Fig. 2).

What may be called its "uniformity" is thus an attractive feature of the spectral centroid scale. On the other hand it is to be remembered that λ_c is dependent upon the particular set of values assumed for the visibility, V_λ. A spectral centroid scale can only be considered "standard" if based on a set of "standard" values of visibility which must

[15] These values apply only to certain experimental conditions. Absolutely smaller probable errors can probably be obtained under improved conditions of observation.

be specified in order to give the scale any significance. Insofar as only illuminants conforming closely to the Planckian distribution law are concerned, it is *not*, however, necessary that the observer's individual visibility be standard in order to obtain standard values of λ_c by way of observing ϕ (or ϕ_2). In case the spectral distribution of the energy being investigated (e.g. "artificial daylight" by the blue filter method) is not Planckian and the observer's visibility is not standard, he may (at least in theory) obtain values of λ_c (and also T) which are not standard. (Cf. discussion of color stimuli of Class 2 under section II-1 above.) In our work, the spectral centroid scale is based on the values of visibility given in column 2 of Table 5. (Appendix.)

Fig. 2 will serve as a condensed summary of the more important available data on the quality of illuminants as indicated by color temperature or the spectral centroid of light. Data marked (*) are from the author's own observations by the rotatory dispersion method. Other more or less standard data culled from the literature have also been inserted to fill out the scales and show the relative positions of various common illuminants and standard sources.

III. THE MEASUREMENT OF INTENSITY

1. The Problem of the Photometry of Illuminants having Different Qualities of Color.

a) The Essential Difficulties.

The difficulty encountered in the determination of the relative intensity of illuminants having different qualities of color is twofold:—

(1) In attempting to make a brilliance match between the two halves of a photometric field which differ in quality, the observer is confused and his judgment rendered uncertain. He is conscious of this; he finds great difficulty in deciding whether or not the field is matched; and consequently such observations are notably lacking in precision.[16]

(2) Besides this observational difficulty, of which the observer is conscious, there exists a "personal equation" of which he is quite unconscious. This "personal equation" lies in the fact that (whether it be due to differences in their ocular media, their retinas, or their brains) individuals do differ notably in the relative visual value which they ascribe to radiant energy of different wave-lengths. These

[16] It is worthy of note that "color blind" observers, that is those whose discrimination of quality of color is not keen, make intensity measurements by way of brilliance match with a precision better than normal.

individual characteristics are set forth very completely and in great detail in the results of numerous researches on the "visibility of energy." This second difficulty results not in low precision of any one observer's observations but in notable differences between the results of different individuals.

The first difficulty is met with great satisfaction experimentally by the use of the flicker photometer, which affords a criterion appearing much more definite to the observer, and by which he may make observations approaching in precision those which he can make by a brilliance match with no quality difference. However, although the flicker photometer may modify ("for better or worse") the personal equation due to the individual visibility, it does not in any wise eliminate it. The outstanding fact is that the relative intensities of illuminants of different quality are notably different for some observers from what they are for other observers. There can, therefore, be no generally valid concept of the "correct" value for the ratio of the intensities of two illuminants of different quality, unless by "correct" we mean an average value (under specified conditions) representing the whole population considered statistically. It will probably be readily granted by all conversant with the subject that the paramount object of photometry is the determination of the "correct" value as thus defined. Although there will be general agreement as to the *intent* of this definition of the correct value, it is obvious that to be applied, it must be supplemented by a more explicit definition in technical terms, the effect of which must be to specify *how* this statistical average is to be determined. The definition being settled, there still remains the question of a suitable routine method of measurement designed to give results in terms of the definition. We shall deal with both of these questions in what follows:

b) The General Practical Solution in Terms of Standard Values of Visibility and the Theorem of the Additivity of Homogeneous Lights.

If I denote the luminous intensity of any source of heterogeneous radiant energy and I_λ denote the luminous intensity of the same source as a function of wave-length, the Theorem of the Additivity of Homogeneous Lights states that

$$I = \int_0^\infty I_\lambda d\lambda \qquad (8)$$

Or, in words: *The observed luminous intensity due to the joint action of all of the homogeneous components is equal to the sum of the separately observed intensities of these same components.*

This law has been verified by H. E. Ives and others (under specified conditions), is in accord with extended experience in photometric work and may be regarded as sufficiently well established to serve as the basis of *a definition of relative luminous intensity for practical purposes*, even if still subject to investigation as regards minor secondary effects. Such a definition, to be formulated immediately below, is assumed in what follows.

If $(E_\lambda)_s$ denote radiant energy, as a function of wave-length, emitted by a source of standard luminous intensity, I_s, and $(E_\lambda)_x$ have like meaning for a source of unknown luminous intensity, I_x, then, in accord with the above mentioned additivity theorem, the definition of relative luminous intensity is

$$\frac{I_x}{I_s} \equiv \frac{\int_0^\infty V_\lambda (E_\lambda)_x d\lambda}{\int_0^\infty V_\lambda (E_\lambda)_s d\lambda} \tag{9}$$

and the unknown luminous intensity I_x is given as

$$I_x = I_s \left(\frac{\int_0^\infty V_\lambda (E_\lambda)_x d\lambda}{\int_0^\infty V_\lambda (E_\lambda)_s d\lambda} \right) \tag{10}$$

We now proceed to apply this definition to the practical determination of I_x. Let the two halves of a photometric field be illuminated respectively by the standard source and a suitable comparison source (a lamp similar to the standard). Let the comparison source be adjusted until the field is matched as regards both brilliance and quality, which will make the spectral distribution of energy in the comparison field the same as in the standard field. Maintaining the comparison source constant, substitute the source of unknown intensity for the standard. Match the field again, as regards both brilliance and quality, by means of two independent adjustments, viz:

(1) One which reduces the brightness of the comparison field by a factor t_λ, which is a function of wave-length.

(2) Another which changes the brightness of the comparison field by a factor k (or changes the brightness of the test field by a factor $\frac{1}{k}$), which is independent of wave-length.

It may then be readily seen that

$$\frac{I_x}{I_s} = \frac{k \int_0^\infty t_\lambda V_\lambda (E_\lambda)_s d\lambda}{\int_0^\infty V_\lambda (E_\lambda)_s d\lambda} \tag{11}$$

The rotatory dispersion colorimetric photometer provides the means of making the two above mentioned adjustments; and the theory of it provides the means of evaluating the factor

$$\frac{\int_0^\infty t_\lambda V_\lambda (E_\lambda)_s d\lambda}{\int_0^\infty V_\lambda (E_\lambda)_s d\lambda} \tag{12}$$

in terms of

(1) *The constants of the apparatus* (which are strictly reproducible from specifications);

(2) *The radiant energy of the standard source as a function of wave-length;*

(3) *The visibility of radiant energy as a function of wave-length.*

How this is accomplished will be explained more concretely in the next section.

2. THE MEASUREMENT OF RELATIVE INTENSITY BY THE ROTATORY DISPERSION PHOTOMETER.

The experimental procedure for the measurement of relative intensity is identical with that described for the measurement of color temperature (II-4 above, the particular case being determined by the color temperature of the unknown) with the following additional provisions:

(1) While adjusting the comparison source to match the standard, the quartz plates are replaced by glass plates of the same transmission.

(2) The adjustment to match the unknown is made by such means that the factor k (III-1 above) is known.

(3) The values of ϕ (or ϕ_1 and ϕ_2 in case the duplex filter is used) which obtain when the comparison field matches the standard as well as when it matches the unknown are noted.

We adopt symbols as follows:—

For the case of the simple filter,

ϕ_s is the value of ϕ when the standard is matched;

ϕ_x is the value of ϕ when the unknown is matched.

For the case of the duplex filter,

$\phi_{1,s}$ is the value of ϕ_1 when the standard is matched;

$\phi_{1,x}$ is the value of ϕ_1 when the unknown is matched;

$\phi_{2,s}$ is the value of ϕ_2 when the standard is matched;

$\phi_{2,x}$ is the value of ϕ_2 when the unknown is matched.

Referring to formulas 2 and 3 for the relative spectral transmissions of the simple and duplex filters, it will be apparent that t_λ in equation 12 may be replaced by $\sin^2(\phi_x - 0.5a_\lambda)$ or $\sin^2(170° - 0.5a_\lambda)\sin^2(\phi_{2,x} - 0.5a_\lambda)$ according as use is made of the simple or the duplex filter.

For color temperatures of the unknown below about 4000° K, using the simple filter, we then have

$$\frac{I_x}{I_s} = \frac{k}{\sin^2\phi_s}\left[\frac{\int_0^\infty V_\lambda (E_\lambda)_s \sin^2(\phi_x - 0.5a_\lambda) d\lambda}{\int_0^\infty V_\lambda (E_\lambda)_s d\lambda}\right] \tag{13}$$

For color temperatures of the unknown above about 4000° K using the duplex filter we have

$$\frac{I_x}{I_s}=\frac{k}{\sin^2\phi_{1,s}\sin^2\phi_{2,s}}\left[\frac{\int_0^\infty V_\lambda(E_\lambda)_s\sin^2(170-0.5a_\lambda)\sin^2(\phi_{2,x}-0.5a_\lambda)d\lambda}{\int_0^\infty V_\lambda(E_\lambda)_s d\lambda}\right] \quad (14)$$

To the photometrician interested only in the practical application of this method, the forbidding parts of these formulas will be the factors

$$\frac{\int_0^\infty V_\lambda(E_\lambda)_s\sin^2(\phi_x-0.5a_\lambda)d\lambda}{\int_0^\infty V_\lambda(E_\lambda)_s d\lambda}$$

and

$$\frac{\int_0^\infty V_\lambda(E_\lambda)_s\sin^2(170-0.5a_\lambda)\sin^2(\phi_{2,x}-0.5a_\lambda)d\lambda}{\int_0^\infty V_\lambda(E_\lambda)_s d\lambda}$$

For his comfort, we may say at once that these factors have already been evaluated for all cases and put in tabular form. They are in general, denoted by the symbol R, so that equation 13 reduces to

$$\frac{I_x}{I_s}=\frac{kR}{\sin^2\phi_s} \quad (15)$$

and equation 14 reduces to

$$\frac{I_x}{I_s}=\frac{kR}{\sin^2\phi_{1,s}\sin^2\phi_{2,s}} \quad (16)$$

where k may be determined by any usual photometric method such as variation of distance, rotation of a nicol prism or use of a sectored disk, and the value of R, pertaining to the standard source used and the observed values of ϕ_x (or $\phi_{2,x}$), may be read directly from tables with the same ease with which logarithms or trigonometric functions may be obtained. There are four sets of tables in which R is shown as a function of ϕ (or ϕ_2) (See Appendix). These tables correspond to the following conditions:

(1) Standard source at color temperature 2077° K, using the simple filter. (Table 1.)

(2) Standard source at color temperature 2360° K, using the simple filter. (Table 2.)

(3) Standard source at color temperature 2848° K, using the simple filter. (Table 3.)

(4) Standard source at color temperature 2848° K, using the duplex filter. (Table 4.)

The preparation of these tables has of course involved the assumption of standard data for:

(1) The relative spectral distribution of energy (E_λ), for each of the standard sources;

(2) The relative spectral distribution of visibility.

Relative to the spectral distribution of energy for the standard sources, nothing need now be said, except to remark that, on the basis of standardization at both the Nela Research Laboratories and the Bureau of Standards, these sources are now known and may be reproduced in this respect, with an accuracy exceeding that required by the problem. (Cf. e. g. B. S. Sci. Pap. 443.)

We shall consider the selection of standard visibility data in the following section.

3. Discussion of Visibility Data and the Choice of Standard Values of Visibility.

As is well known, the relative visibility of energy as a function of wave-length has been the subject of numerous extensive and thoroughgoing experimental investigations, particularly during the past ten years. A graphic synopsis of the more important data is shown in Fig. 4.

It is of prime importance to our argument to observe that *the differences between the values recommended by different investigators using various methods and different observers are quite small relative to the differences between individuals found by Coblentz and Emerson whose investigation comprehended more observers than any of the others.* (See Fig. 4.) From this we infer that the values adopted by the I. E. S. Committee or the values adopted by the present author represent the "true" statistical values as accurately as it is feasible to determine them considering the range of variation among individuals. The recent results of Gibson and Tyndall[17] confirm this conclusion.

Classifying the results of different investigations in general with respect to the ratio of visibility for greater wave-lengths to the visibility for lesser wave-lengths, we find that the results of Hyde, Forsythe and Cady on one hand and Coblentz and Emerson on the other represent the extreme cases. Now, we have computed full tables of the factor R not only for the assumed standard values of V_λ as given in columns 2 and 3 in Table 5 but also for the extreme cases of Hyde, Forsythe, and Cady on one side and Coblentz and Emerson on the other as well as for the intermediate values recommended by various other

[17] B. S. Sci. paper No. 475. Preliminary announcement of results, Jo. Opt. Soc. Am. 7, pp. 68-69; 1923.

investigators.[18] An abridgment of these tables is given in Tables 1, 2, 3, 4. Reference to these tables will suffice to show that:—

(1) Even the extreme difference introduced into measurements of relative luminous intensity by assuming for visibility, on one hand the data of Hyde, *et al* and, on the other the data of Coblentz and Emerson, would be hardly greater than that usually considered unavoidable and negligible (one to two percent) in all photometric work

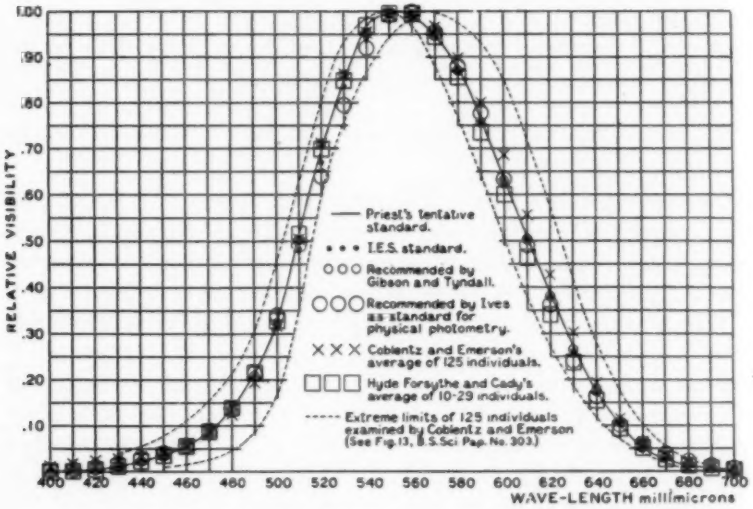

RELATIVE VISIBILITY
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
400 420 440 460 480 500 520 540 560 580 600 620 640 660 680 700
WAVE-LENGTH millimicrons
Priest's tentative standard.
I.E.S. standard.
Recommended by Gibson and Tyndall.
Recommended by Ives as standard for physical photometry.
Coblentz and Emerson's average of 125 individuals.
Hyde Forsythe and Cady's average of 10-29 individuals.
Extreme limits of 125 individuals examined by Coblentz and Emerson (See Fig.13, B.S.Sci Pap. No. 303.)

FIG. 4. *Synopsis of visibility data* (*Cf. Tab. 5, Appendix*)

The data assumed as standard in the present paper are shown by the continuous curve.
References:

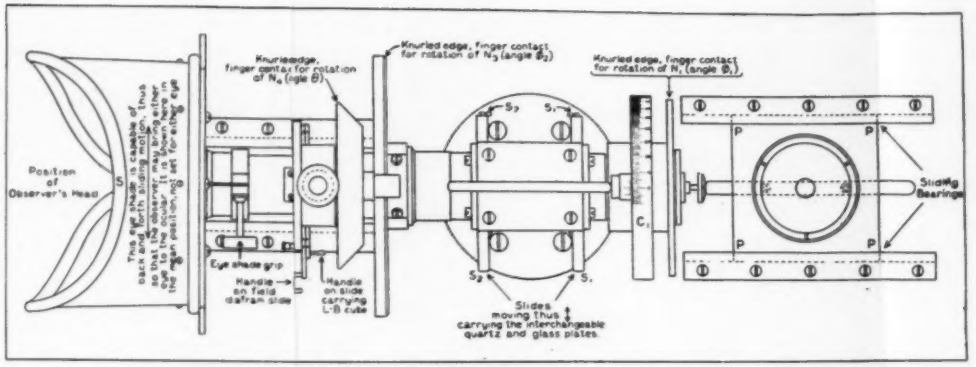
I. E. S. Com. on Nomenclature and Standards, Trans. I. E. S. *13*, p. 523; 1918.
Gibson and Tyndall, B. S. Sci. Pap. No. *475*, last col. of Table 3; 1923.
H. E. Ives, Jo. Frank. Inst., 188, p. 220, Tab. I; 1919.
Coblentz and Emerson, B. S. Sci. Pap. No. *303*, Tab. 5 and Fig. 13; 1917.
Hyde, Forsythe and Cady, Astrop. Jour. *48*, p. 87, Tab. IV, 1918.

except that of the utmost refinement, even when no quality difference is involved.

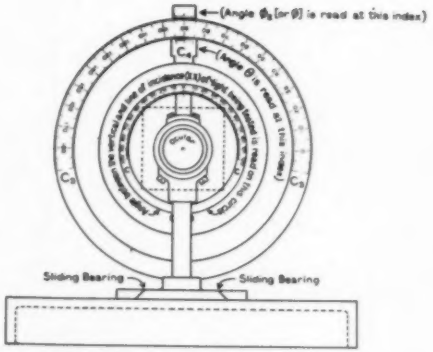
(2) The difference introduced by assuming the I. E. S. standard values on one hand and those adopted by the present author, or any other reasonable statistic average, on the other will be entirely negligible (one to two tenths of one percent) as compared with the accuracy of

[18] These intermediate results which cannot be considered in this paper will be dealt with fully in a forthcoming B. S. Sci. Pap. under the same title.

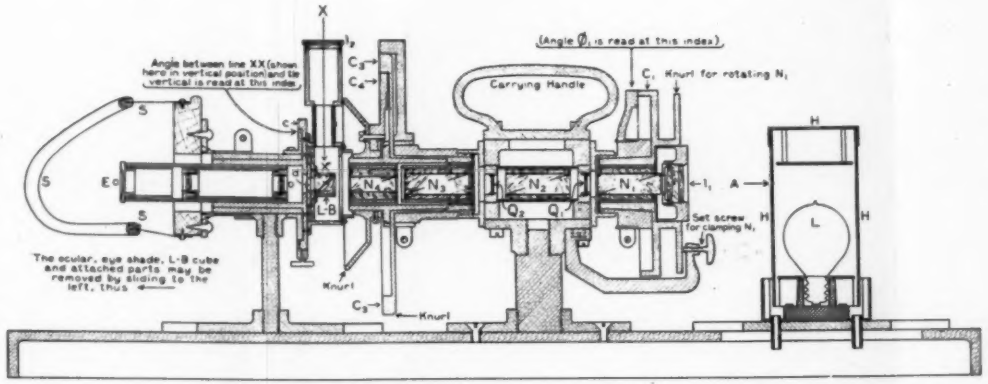
N_1 N_2 N_3 N_4 Nicol prisms.
Q_1 Q_2 Quartz plates, each 0.500mm thick, mounted in slides (S_1 and S_2) and interchangeable with glass plates, in same slides.
L-B Lummer-Brodhun cube, removable by slide.
E-A Line of sight, optic axis of quartz, axis of rotation of L-B, N_1, N_2, N_3, N_4.
C_1 Circle for rotating N_1, and reading its rotation relative to N_2 (ϕ_1)
C_3 Circle for rotating N_3, and reading its rotation relative to N_2. (ϕ_2)
C_4 Circle for rotating N_4, and reading its rotation relative to N_3. (θ)
c Circle for rotating L-B.
o-o Ocular.
L Comparison lamp. P Sliding plate carrying L and H.
H White lined ventilated housing for L.
A Aperture in H.
l_1 Lens forming image of A in L-B.
X-X Line of incidence of light to be examined. (By rotating L-B about E-A this line may be made vertical or horizontal or brought to *any* desired elevation.)
d Field diaphragm. (Interchangeable diaphragms of different sizes are provided in a slide to give field sizes of 2°, 3° and 6°.)
l_2 Lens.
S Eye shade.


Position of Observer's Head
Knurled edge, finger contact for rotation of N_4 (angle θ)
Knurled edge, finger contact for rotation of N_3 (angle ϕ_2)
Knurled edge, finger contact for rotation of N_1 (angle ϕ_1)
Eye shade grip
Handle on field diaphragm slide
Handle on slide carrying L-B cube
Slides moving thus ↕ carrying the interchangeable quartz and glass plates
Sliding Bearings

PLAN



FRONT ELEVATION
(with eye shade removed)


X
(Angle ϕ_1 is read at this index)
Angle between line XX (shown here in vertical position) and its vertical is read at this index
Carrying Handle
C_1 Knurl for rotating N_1
The ocular, eye shade, L-B cube and attached parts may be removed by sliding to the left, thus ←
Knurl
Knurl
Set screw for clamping N_1

VERTICAL MEDIAN SECTION

FIG. 5. *Rotatory dispersion colorimetric photometer. Construction plans: scale; 1cm = 3.5cm.*

photometry of the highest precision when no quality difference is involved.

The following conclusions therefore seem to be justified:

(1) The visibility function is sufficiently well determined to serve as a fundamental standard in the photometry of incandescent sources.

(2) No reasonable statistical average of the available data on visibility could depart from the I. E. S. standard to a sufficient amount to introduce any appreciable difference in the resultant relative luminous intensity of incandescent sources as derived from this standard.

(3) In view of the extensive data already available on relative visibility, it is very unlikely that the results of any future investigation will alter the reasonable statistical values to an amount sufficient to affect the photometry of incandescent sources.

(4) At present, in the application of the rotatory dispersion method, it will make absolutely no difference in practical results whether the photometrician assumes, in theory, the visibility values adopted by the I. E. S. Committee or the values independently chosen by the author.

However, the tables of values of R having already been prepared on the basis of most widely different values of V_λ, any individual, in applying the method, may choose any values which he likes as standard. Further, if any revision of the values of V_λ is contemplated as bearing on the photometry of incandescent sources, it may be suggested that this method, used in conjunction with others, may serve as a valuable tool in determining the most probable effective values.

IV. THE INSTRUMENT AND ITS USE

The purposes of this section of the paper are to describe the particular instrument which has been built, and to give explicit instructions for its use in the determination of color temperature and relative luminous intensity. It is addressed particularly to those who may be interested only in the practical application of the instrument as a tool, who may perhaps be willing to take the theory of it on faith, but desire definite working instructions.

The construction plans of the instrument which has been built are shown in Fig. 5. A perspective view of it is shown in Fig. 6. These figures are, to a large degree, self explanatory, and careful study of them should lead to rather definite ideas of the use of the instrument without the need of very extended explanation.

This particular instrument has certain incidental features which are not essential to the rotatory dispersion method in general. These are:

(1) The measurable rotation of the Lummer-Brodhun cube about the axis *E-A* by the circle *c* in order to point at the sky at different elevations in determinations of the brightness and color of the sky.

(2) The measurable rotation of the whole instrument in azimuth about the vertical axis of the supporting pillar, (see Fig. 6).

(3) The small comparison lamp, (automobile headlight lamp) attached as a part of the instrument to make the whole a conveniently portable unit for field work.

The lamp housing *H* may be lifted off by loosening the three small thumb screws at the base (two of which show in Fig. 6). Since the line of sight *E-A* (Fig. 5) is above the lamp *L*, a clear view of any other desired comparison source may thus be obtained without removing the comparison lamp (*L*, Fig. 5) from its socket.

(4) The additional nicol, N_4, which, by rotation relative to N_3, (angle θ) serves to vary brilliance independently of quality in the comparison field.

The arrangement of the essentially characteristic parts, N_1, N_2, N_3 and Q_1, Q_2, and *L-B* corresponds to that already shown in Fig. 3.

On looking into the instrument, one sees a circular photometric field divided on a diameter by the Lummer-Brodhun cube. There are three interchangeable field diaphragms in a slide, giving fields the diameter of which subtend respectively 2°, 3°, and 6° at the observer's eye. One half of the field is illuminated by light traveling in the direction *A-E* from the comparison source. That is, one half of the field is illuminated by light reflected from the interior white wall of the comparison lamp housing. The other half is illuminated by light incident on the Lummer-Brodhun cube along the line *X-X* which may be pointed in various directions by turning the circle *c*.

Instructions[19]

Zero adjustment of nicol prisms. Both quartz plates and the Lummer-Brodhun cube being removed from the line of sight by slides which provide for this, the observer looks through the instrument in the direction *E-A* towards a very bright source, and (with the other nicols set for roughly maximum transmission) separately sets each index

[19] It is assumed that the reader is already familiar with the ordinary use of a pair of nicol prisms in photometry.

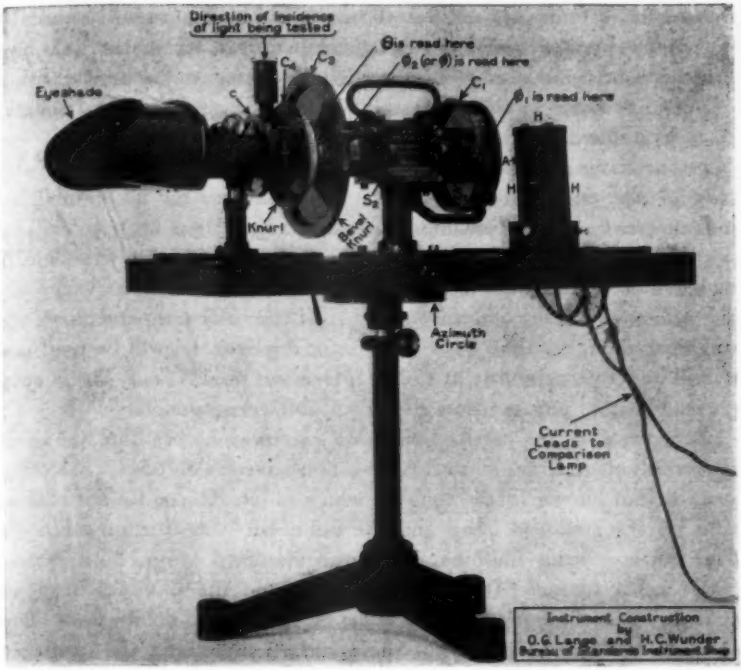


FIG. 6. *Rotatory dispersion colorimetric photometer*
Perspective View
(For significance of symbols, see legend, Fig. 5).

The instrument is here shown as set for observations on light incident on the Lummer-Brodhun cube in a vertical line from above, as it would be used, for example, in observations on the sky at the zenith. However, it may be set (as shown by construction in Fig. 5) for incidence in a horizontal or any intermediate direction. The whole instrument with all parts, may also be rotated about the axis of the supporting pillar and its azimuth read on the divided circle just under the base plate.

In use, the simultaneous and independent variation of quality and brilliance in the comparison field may be attained by the observer placing his fingers on the back side of circle C_3 at the point marked "bevel knurl" in this view, and his thumbs on the knurled edge of the saucer-shaped circle C_4. Rotation of C_4 relative to C_3 (angle θ) varies brilliance only. Rotation of C_3 and C_4 together as a unit (angle ϕ or ϕ_2) varies both brilliance and quality. By the motion of his hands and the differential motion of his fingers and thumbs, the observer is thus able to match the field for both quality and brilliance very conveniently.

(at which the angles ϕ_1, ϕ_2, or ϕ and θ are to be read subsequently) so that the reading is zero when the illumination of the field is extinguished. (It is even better to merely determine the zero readings for extinction and apply them later as corrections to the observed values of ϕ_1, ϕ_2, ϕ and θ).

Selection of standard source. The choice of a standard source is determined by the range of color temperature in which the unknown is expected to fall and will be settled by reference to tables 1 to 4 (Appendix) showing the applicability of the several standard sources to measurements of various ranges of color temperature.

Use or disuse of quartz plate Q_1 and setting of ϕ_1. If the color temperature of the unknown is less than 4200° K, i.e., for all unmodified incandescent sources (including the ordinary carbon arc), Q_1 will be permanently removed from the path *E-A* by means of the slide in which it is mounted. (Rotation of N_1 will then serve to vary only the brilliance of the comparison field.) If the color temperature of the unknown is higher than 4200° K i.e., for daylight, Q_1 will be used and ϕ_1 will be set permanently at 170°. If Q_1 is not used N_1 may be clamped permanently in any position giving suitable transmission.

Adjustment of the comparison lamp current. Direct the line *X-X* (by rotation of circle *c*) toward the non-selective diffusely reflecting surface (not shown in the figures) which is later to be illuminated in turn by the standard lamp and the unknown (substitution method). This surface being illuminated by the standard lamp, both quartz plates being removed from the path *E-A* and the Lummer-Brodhun cube being in the path *E-A*, vary the current in the comparison lamp so as to match the quality of the standard, matching the brilliance simultaneously by rotation of N_4 relative to N_3, or N_3 relative to N_2, or both. Several readings of this current should be made and the mean taken, to be thenceforth maintained constant in the comparison lamp.

Observation of data to be used in computing color temperature and relative intensity. The above mentioned constant current being maintained in the comparison lamp, the standard lamp being in position, and the glass plates (interchangeable with the quartz plates) being in the path *E-A*, the field is matched by rotation of N_4 relative to N_3 (angle θ) or N_3 relative to N_2 (angle ϕ) or both and the angles ϕ (or ϕ_2 if both quartz plates are to be used) and θ are read.

The unknown is now substituted for the standard, the quartz plate Q_2 (or the plates Q_1 and Q_2 if the color temperature of the unknown is over 4200° K) is returned to position in the path *E-A*, and a match of quality of color is produced by rotation of N_3 relative to N_2 while a brilliance match is simultaneously made by rotation of N_4 relative to N_3. These values of ϕ (or ϕ_2) and θ are read.

Computation. *The color temperature* (or spectral centroid if desired) of the unknown is obtained from the observed value of ϕ (or ϕ_2) by direct reference to the tables or curves showing T or λ_c as functions of ϕ (or ϕ_2).

The intensity of the unknown luminous flux is

$$I_x = \frac{I_s R \sin^2\theta_x}{\sin^2\phi_s \sin^2\theta_s} \tag{17}$$

if only one quartz plate has been used; or

$$I_x = \frac{I_s R \sin^2\theta_x}{\sin^2\phi_{1,s} \sin^2\phi_{2,s} \sin^2\theta_s} \tag{18}$$

if both have been used.

I_s is the intensity of the standard source.

The subscripts s and x indicate respectively values of the angles observed in matching the standard and the unknown.

R is read as a function of ϕ_x (or $\phi_{2,x}$) from the table or curve corresponding to the standard source which was used (See Appendix).

Auxiliary photometric readings. It may be necessary to reduce the illumination from the unknown in order to secure a match. This may of course be done by any of the usual methods (increase of distance, sector disk, etc.) and a factor taking account of the change entered in equation 17 or 18.

Special applications. In the preceding pages on photometry, we dealt formally only with measurements of the relative luminous intensity of incandescent sources. The applicability of the instrument to measurement of the brightness of surfaces and "sky brightness" is so obvious as to hardly need mention. Another use which will also be obvious on a moment's thought is the measurement of the total light transmission (for sources at given color temperatures) of "daylight glass" and other blue and yellow filters now used in many laboratories to "color match" illuminants as a step in the measurement of relative intensity.

V. CONCLUSION

This method and instrument provide for the specification of color quality and the measurement of relative intensity for all incandescent illuminants and the important phases of daylight, including, on one extreme, the Hefner lamp, and, on the other, the blue sky.

The practical solution of the problem of the photometry of illuminants of different quality here proposed is given explicitly in the most simple and fundamental terms possible in the nature of the problem. The method is free of non-reproducible terms and temporary expedients in specification.

At the same time it appears to be convenient and well adapted to ordinary routine work in the laboratory or in the field.

This method is therefore recommended as a fundamental and convenient solution of the problems of the colorimetry and photometry of incandescent sources and daylight.

NATIONAL BUREAU OF STANDARDS,
SEPTEMBER, 1923.

APPENDIX

TABLES OF COLOR TEMPERATURE (T), SPECTRAL CENTROID OF LIGHT (λ_c), AND THE PHOTOMETRIC FACTOR, R, AS FUNCTIONS OF ϕ AND ϕ_2 FOR THE THREE STANDARD SOURCES.

The tables given here are an abridgment of much more detailed ones to be published later in the Bureau of Standards Scientific Paper on this subject. By these fuller tables, it will be possible to read T, λ_c and R generally with sufficient accuracy *without interpolation*.

Altho these abridged tables are given here primarily for illustration rather than for use, they cover the complete range of the method as described, and practical use can be made of them by using them merely as data for plotting curves which can then be used to find T, λ_c, or R for any given value of ϕ or ϕ_2.

The data on which the tables are based, the methods of constructing them, and their accuracy will also be discussed in connection with their publication in full.

The author desires to acknowledge the co-operation of his associates, Dr. K. S. Gibson, Dr. M. Katherine Frehafer and Mr. H. J. McNicholas as well as the routine assistance of several computers in various computations extending over a period of years which have contributed to the production of these tables in their present form.

The very great labor of actual numerical computation of the final tables as given here has been carried out in a careful and painstaking manner by Messrs. J. Keith Cromer and F. A. Durr. The final results have, however, been checked both experimentally and by computation to such an extent that the author feels justified in assuming responsibility for their correctness.

TABLE 1. *Standard Source Color Temp. 2077°K, Simple Filter*

Visibility→ $\phi\downarrow$	R				λ_e	Color Temperature
	Hyde, Forsythe and Cady	Priest's Tentative Standard	I. E. S. Standard	Coblentz and Emerson	Priest's Tentative Standard	
					mμ	deg. K
40°	0.2292	0.2300	0.2303	0.2311	585.21	1838
45	.3061	.3070	.3073	.3082	584.48	1876
50	.3889	.3899	.3902	.3912	583.91	1906
55	.4750	.4760	.4764	.4774	583.46	1931
60	.5620	.5630	.5633	.5644	583.08	1953
65	.6470	.6480	.6483	.6493	582.76	1971
70	.7276	.7285	.7288	.7297	582.45	1989
75	.8013	.8021	.8024	.8032	582.19	2005
80	.8658	.8665	.8668	.8674	581.95	2019
85	.9192	.9198	.9199	.9205	581.73	2033
90	.9599	.9603	.9604	.9608	581.52	2045
95	.9866	.9868	.9869	.9871	581.30	2059
100	.9985	.9985	.9986	.9986	581.10	2072
105	.9953	.9951	.9951	.9950	580.90	2085
110	.9770	.9767	.9766	.9763	580.69	2099
115	.9442	.9437	.9436	.9431	580.48	2113
120	.8979	.8973	.8971	.8965	580.26	2127
125	.8395	.8388	.8385	.8378	580.03	2143
130	.7709	.7700	.7697	.7688	579.78	2160
135	.6939	.6930	.6927	.6918	579.51	2179
140	.6111	.6101	.6098	.6088	579.19	2202
145	.5249	.5239	.5236	.5226	578.84	2228
150	.4380	.4370	.4367	.4356	578.41	2259
155	.3530	.3520	.3517	.3507	577.90	2297
160	.2724	.2715	.2712	.2703	577.21	2350
165	.1987	.1979	.1976	.1968	576.32	2418
170	.1342	.1335	.1332	.1326	575.07	2530
175	.0807	.0802	.0800	.0795	573.12	2719

TABLE 2. *Standard Source Color Temp. 2360°K, Simple Filter*

Visibility→ $\phi\downarrow$	R				λ_c	Color Temperature
	Hyde, Forsythe and Cady	Priest's Tentative Standard	I.E.S. Standard	Coblentz and Emerson	Priest's Tentative Standard	
					mμ	deg. K
40°	0.2267	0.2275	0.2278	0.2286	581.42	2052
45	.3033	.3043	.3045	.3055	580.66	2102
50	.3860	.3870	.3873	.3882	580.07	2141
55	.4721	.4731	.4734	.4744	579.59	2174
60	.5590	.5600	.5603	.5613	579.19	2202
65	.6442	.6451	.6454	.6464	578.85	2227
70	.7250	.7259	.7261	.7270	578.55	2249
75	.7989	.7997	.7999	.8007	578.28	2269
80	.8638	.8645	.8647	.8653	578.03	2288
85	.9176	.9181	.9183	.9188	577.80	2305
90	.9587	.9591	.9592	.9596	577.56	2324
95	.9859	.9861	.9862	.9864	577.35	2339
100	.9983	.9983	.9984	.9984	577.14	2356
105	.9956	.9955	.9954	.9953	576.93	2371
110	.9778	.9775	.9774	.9771	576.72	2388
115	.9455	.9451	.9449	.9445	576.50	2404
120	.8997	.8991	.8989	.8983	576.27	2423
125	.8417	.8409	.8407	.8400	576.03	2443
130	.7733	.7725	.7722	.7714	575.77	2466
135	.6967	.6957	.6955	.6945	575.49	2492
140	.6140	.6130	.6127	.6118	575.17	2521
145	.5279	.5269	.5266	.5256	574.80	2554
150	.4410	.4400	.4397	.4387	574.36	2596
155	.3558	.3548	.3546	.3536	573.83	2647
160	.2750	.2741	.2739	.2730	573.15	2716
165	.2011	.2003	.2000	.1993	572.23	2815
170	.1362	.1355	.1353	.1347	570.94	2963
175	.0824	.0819	.0817	.0812	568.93	3227

N. B.—For accurate measurements of color temperatures below 2350° K, use Table 1 rather than Table 2. The agreement between Planckian and rotatory dispersion spectral distributions is imperfect for the lower temperatures in Table 2.

TABLE 3. *Standard Source Color Temp. 2848°K, Simple Filter*

Visibility→ ϕ↓	R				λ_e	Color
	Hyde, Forsythe and Cady	Priest's Tentative Standard	I.E.S. Standard	Coblentz and Emerson	Priest's Tentative Standard	Temperature
					mμ	deg. K
140°	0.6179	0.6169	0.6166	0.6158	569.96	3087
145	.5319	.5309	.5306	.5297	569.57	3140
150	.4449	.4439	.4436	.4428	569.12	3201
155	.3596	.3587	.3584	.3576	568.57	3280
160	.2786	.2777	.2775	.2767	567.86	3386
165	.2043	.2035	.2033	.2026	566.93	3537
170	.1390	.1383	.1381	.1376	565.60	3777
175	.0846	.0841	.0840	.0835	563.54	4211

TABLE 4. *Standard Source Color Temp. 2848°K, Duplex Filter*

Visibility→ ϕ_1↓	R				λ_e	Color
	Hyde, Forsythe and Cady	Priest's Tentative Standard	I.E.S. Standard	Coblentz and Emerson	Priest's Tentative Standard	Temperature
					mμ	deg. K
140°	0.0866	0.0860	0.0858	0.0854	563.54	4208
145	.0746	.0741	.0740	.0736	563.15	4308
150	.0625	.0621	.0620	.0616	562.67	4430
155	.0507	.0503	.0502	.0499	562.10	4579
160	.0394	.0390	.0390	.0387	561.36	4800
165	.0290	.0287	.0287	.0285	560.40	5114
170	.0198	.0196	.0196	.0194	559.03	5620
175	.0122	.0120	.0120	.0119	556.92	6710
180	.0063	.00618	.00616	.0061	553.14	10270
182		.00436			550.65	16700
182.5					549.87	21280
182.6					549.71	22500
182.71					549.52	24000

TABLE 5. *Relative Visibility*

λ	1. Hyde, Forsythe and Cady	2. Priest's Tentative Standard	3. I.E.S. Standard	4. Coblentz and Emerson
400mμ	0.00009		0.0004	0.010
10	.00062	0.0006	.0012	.017
20	.0041	.0041	.0040	.024
30	.0115	.0115	.0116	.029
40	.022	.022	.023	.033
450	.036	.036	.038	.041
60	.055	.055	.060	.056
70	.087	.087	.091	.083
80	.138	.138	.139	.125
90	.216	.216	.208	.194
500	.328	.328	.323	.316
10	.515	.515	.484	.503
20	.698	.698	.670	.710
30	.847	.847	.836	.862
40	.968	.968	.942	.954
550	.996	.996	.993	.994
60	.995	.990	.996	.998
70	.944	.948	.952	.968
80	.855	.875	.870	.898
90	.735	.763	.757	.800
600	.600	.635	.631	.687
10	.464	.509	.503	.557
20	.341	.387	.380	.427
30	.238	.272	.262	.302
40	.154	.175	.170	.194
650	.094	.104	.103	.115
60	.051	.051	.059	.0645
70	.026	.026	.030	.0338
80	.0125	.0125	.016	.0178
90	.0062	.0062	.0081	.0085
700	.0031	.0031	.0041	.0040
10	.0015	.0015	.0021	.00203
20	.00074	.00074	.0010	.00097

References

Col. 1. Astrop. Jour. *48*, p. 87, Tab. IV; 1918

2. These values were adopted by Priest as a tentative standard for use in the Colorimetry Section, Bureau of Standards before he was advised of the recommendations of the I.E.S. Com. First published, Jo. Op. Soc. Am. *4*, p. 471, Fig. 8; 1920.

3. Trans. I.E.S. *13*, p. 523; 1918. Ill. Eng. Nomenclature and Standards, Am. Stand. App. July 11, 1922, Am. Eng. Standa. Com., pp. 11-12.

4. B.S. Sci. Pap. No. *303*, p. 219 Tab. 5; 1917.

Recent Study of the Aurora and of the Nocturnal Light of the Sky.—A very delicate measurement of the wave length of the "aurora green line," which constitutes much of the light of the night sky, has lately been made by H. D. Babcock at Mount Wilson. He placed a Fabry-Perot interferometer (translucent gold films on two parallel glass surfaces 1 mm apart) in front of an ordinary hand-magnifier lens, and a green-sensitive photographic plate behind the lens; pointed it at the sky on a moonless night and left it for 10 hours, and without any other apparatus obtained a distinct set of interference rings on the plate. Perfecting the apparatus and separating the parallel surfaces by several cm, he obtained narrow sharp rings, from which it is deduced that the wave length of the line is 5577.350A (with a very small uncertainty in the last place) and its width about .035A. This is about the apparent width which would be imparted by Doppler effect to a truly monochromatic radiation emitted by He atoms at a temperature of 218°K, which he estimates as the temperature of the region where the auroral line is supposed to be emitted. Lord Rayleigh photographed the spectrum of the night sky, with an optical system of enormous light-gathering power and width of field, obtained at the sacrifice of dispersion; he exposed individual plates for as much as 200 hours (a process naturally requiring a whole season) and obtained the green line, two additional lines in the violet, and the *H* and *K* absorption lines projected against a brighter continuous background. Dufay also made photographs of the spectrum of the night sky, and detected the conspicuous absorption lines of the solar spectrum. These data are to be taken as indications that an appreciable part of the light of the night sky is scattered sunlight.

A theory of the aurora proposed by H. Bongards involves some of the newest advances in atomic physics. It is based on the assumption that the aurora green line is identical with the line 5577.98 in the blue or enhanced spectrum of argon; an assumption which Bongards reinforces by identifying two other lines photographed by Vogel in auroral spectra, in the year 1871, with argon lines at 6297.15 and 5187.47, but which is apparently rendered less acceptable by Babcock's determination of the wave length of the green line. Bongards thinks that the argon atoms consist of calcium nuclei which depart from the sun (calcium being very prominent in the solar protuberances); either lose a proton (becoming the argon isotope of weight 36) or gain two intra-nuclear electrons (becoming the argon isotope of weight 40); shoot through the upper atmosphere at immense speed, without becoming luminous, until they are slowed down enough to gather in orbital electrons from the atmosphere; whereupon they radiate until they reach the end of their range at the bottom of the auroral layer. It is a striking feature of this theory that the two argon isotopes should have different ranges, which may serve to explain the two distinct heights at which the lower boundary of the aurora is observed to occur most frequently; there is even a fair quantitative agreement. The much longer auroral 'rays' sometimes observed may be interpreted as the trails of argon atoms which arrive in the outer atmosphere with their complement of orbital electrons.

[H. D. Babcock, Mount Wilson; Astroph. Jl. *57*, 209-221; 1923. Lord Rayleigh; Proc. Roy. Soc. *A103*, 45-52; 1923. Dufay; C. R. *176*, 1290-1292; 1923. H. Bongards; Phys. ZS. *24*, 279-285; 1923.]

K. K. DARROW

DIFFUSION OF LIGHT FROM A SEARCHLIGHT BEAM*

By Enoch Karrer and U. M. Smith

Contents

		Page
I.	Definition and general importance of diffusion of light	1211
II.	Review of general characteristics of light diffused by small particles	1212
	a. Brightness	1212
	b. Polarization	1212
	c. Typical examples	1213
III.	Data from searchlight beams	1214
	a. Brightness	1214
	b. Polarization	1217
	c. Comparison of observations with theory	1219
IV.	Practical importance of light diffused from searchlight beams	1220
	a. Effect on visibility at various distances from the searchlamp	1220
	b. Effect on the range of searchlamps	1223
	c. Effect of eliminating the polarized component from light diffused	1224
V.	Color of the path of various searchlight beams—general considerations	1226
VI.	Bibliography	1230

I. Definition and Importance of Diffusion of Light

Diffusion is the process by which bodies not self luminous become luminous when intercepting beams of light. It is brought about by the deflection of the rays in various directions by refraction, reflection, diffraction or scattering. The diffusion of light by the atmosphere where all of these factors may play a rôle is of great importance in the use of searchlamps.[1] The gas molecules and very fine solid particles will "scatter"; the larger particles will reflect and the water particles will also refract.

As a consequence of diffusion the path of the searchlight beam is made luminous and is a convenient index to the adjustment and orientation of the searchlamp, and may render service as land marks on land and in air, and as lighthouse beacons. In contrast with these advantages, however, we have the fact that light diffused from the searchlight beam interferes with vision when the searchlights are used to pick up targets. On the one hand diffusion means actual loss of illumination of the target; on the other, it virtually interposes luminous matter between the target and the eye and thus obscures the target.

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[1] See discussion by Karrer and Tyndall, Trans. I.E.S. *14*, p. 142, 1919.

The greater the diffusion, the smaller therefore is the contrast between the target and the luminous stretch of atmosphere along the path of the searchlight beam through which the target is viewed. An illustration of an extreme case is when in very dense fogs the disk of the sun becomes obscured and invisible. The manner in which visibility depends upon contrast is not simple,[2] but there are some data available that are applicable to actual searchlight conditions.[3]

II. General Characteristics of Light Diffused by Small Particles

Intensity and Polarization

The amount of light diffused by the air molecules and by very small electrically non-conducting particles theoretically varies inversely as the fourth power of the wave-length of the light.[4] The intensity I of the light diffused is given by $I = \frac{k}{\lambda^4} I_0 (1+\cos^2 B). \frac{m}{r^2}$ where I_0 is the intensity of the incident beam; λ, the wave-length; k, a factor depending upon size and density of the particles; B, the angle between the line of sight and the incident beam; m, number of particles in unit volume; r, the distance from the particle.

When the linear dimensions of the particles become comparable with the wave-length this "scattering" plays a less important rôle and diffusion by reflection becomes more important. The kind of light diffused will then depend almost entirely upon the optical properties of the material of the particles, and the amount of the diffused light upon the total cross-sectional area presented to the beam by the particles. This statement is verified in the differently colored smokes and fumes met with in military and other activities, and in many colloidal solutions.

The light scattered in various directions is polarized to a greater or less degree. The maximum polarization from small spherical particles is in direction perpendicular to the beam of light. For molecules the polarized portion may be nearly 100 per cent. As the size of the particles increases, the direction along which there is maximum polarization for light of any given wave-length shifts towards the direction of propagation of the incident beam,[5] and the degree of polarization decreases. The relative spectral diffusion is then no

[2] Reeves, J. A. Opt. *4*, p. 35; 1920.

[3] Karrer and Tyndall. Contrast Sensibility of the Eye. B.S. Sci., Pap., 366; 1920.

[4] Rayleigh, Phil. Mag. *4*, 41, p. 107; 1871, and 47, p. 375; 1899.

[5] Rayleigh, Phil. Mag., *12*, p. 81, 1881; On the Electromagnetic Theory of Light. Burton, The Physical Properties of Colloidal Solutions, p. 96.

longer as the inverse fourth power, but may be as the inverse eighth power of the wave-length. The effect of wave-length may be the same as the effect due to the size of the particles. Thus it has been shown[6] that for gold particles in colloidal solution up to 100 mμ (100×10^{-9} meter) the maximum polarization is almost 100 per cent in a direction at 90° to the incident beam. For particles of diameter 160 to 180 mμ, the maximum polarization lies between 110° and 120°.

When the diffusing particles are not electrically insulating, the diffusion effects are different.[7] Thus for spherical conducting particles the direction in which the polarization is a maximum makes an angle of 120° with the direction of the incident beam. This result holds when the wave-length of the light is small compared with the product of the ratio of the magnetic permeability to the specific resistance and the square of the radius of the particle;[8] being at the same time large compared with the diameter of the particle.

It should further be borne in mind that the amount of polarization and the direction of maximum polarization depend also upon the shape of the particles. For example, if the particles are cylindrical instead of spherical, the polarization at 90° is less and in directions other than perpendicular may change in various ways, presenting maxima and minima and even reversals depending upon the relative magnitude of the radius of the cylinder and the wave-length of the light.[9] It is clear also that if there is not uniformity of size and shape many variations from these definite relationships are possible.

That there is appreciable diffusion by the molecules alone is evidenced by the blue sky and by the distinct outline of the searchlight beam in the clearest atmosphere and at high altitudes.[10] The red sun, (commonly seen at sunset through a hazy atmosphere, and less frequently during the day through the smoky atmosphere of large cities) is due to diffusion by solid particles. The blurred outline of objects in fogs and rains results from diffusion by water particles.[11]

[6] Mie. Ann. der Phys., *25*, p. 337, 1908; or Burton, p. 111.

[7] J. J. Thomson, Recent Researches, p. 449; and Burton, p. 103.

[8] J. J. Thomson, see ref. 7.

[9] Rayleigh, Phil. Mag. *36*, p. 365; 1918.

[10] This is qualitatively correct. The rigid proof that molecules scatter lies in the laboratory experiments with gases of the highest purity attainable. The atmosphere is so full of dust that there is a constant haze horizon as aviators have witnessed. This dust is certainly more potent than molecules in diffusing light in the lower atmosphere. It was our plan to make some measurements at high altitudes from airplanes in this connection.

[11] Other striking illustrations of diffusion by solid particles where size and shape play important rôles are found in the colors of insects. (Rayleigh, Phil. Mag., 37, p. 98; 1919.)

III. DATA FROM SEARCHLIGHT BEAMS

BRIGHTNESS

The amount of light diffused from the beam affects the visibility in that it determines the brightness of the background against which the target must be viewed.

We have measured the brightness of the path of the searchlight beam by means of the photometer *P* shown in Fig. 1. It was equipped with a tube *F* containing a long focus lens *C*, a mirror, and attached

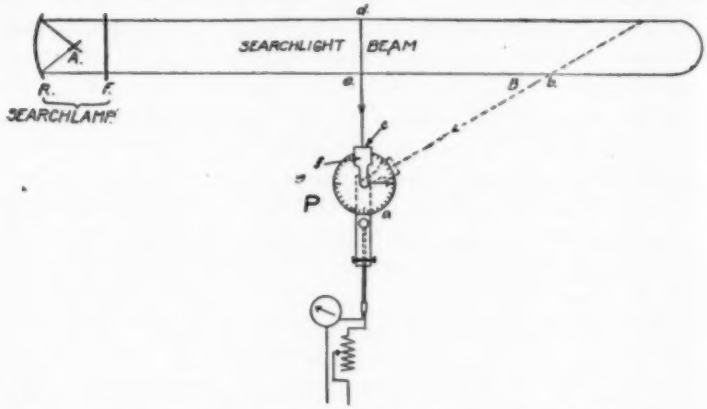

FIG. 1. *Disposition of apparatus for determining brightness of the path of the beam. A, arc; R, reflector; F, front glass; P, photometers; B, angle between line of sight and beam.*

to it a disk graduated to 5 degree intervals. The lens reduced the effective aperture of the photometer to 1°.19; the graduated disk indicated the direction of the line of sight of the photometer relative to the beam; the mirror made it possible to change the line of sight without

Another illustration, where the relative importance of the fineness of the particle is brought out lies in the color of coins composed of copper and nickel. These coins more nearly assume the color of nickel than of copper, although the latter metal is present in far greater proportion than the former. The color of a cross-section of a plate formed electrolytically by alternate deposition of nickel and copper may be more nearly like that of nickel than that of copper, even when the ratio of the quantity of the latter to that of the former is 9 to 1. More common illustrations of this are in dyes and powders. The rainbow and halo as well as the iridescent colors of clouds are also examples of diffusion by water particles, where refraction is a governing factor. (See Rayleigh, Phil. Mag., *47*, p. 404; 1899, On the Transmission of Light through an Atmosphere Containing Small Particles in Suspension and on the Origin of the Blue of the Sky.) Refraction and interference are factors in diffusion of light in a clear atmosphere that, on account of local varying differences in refractive index, give twinkling and shimmering effects. This is sometimes referred to as optical haze.

moving the whole photometer and without changing the observer's position. Measurements were made with the searchlight beam both horizontal and vertical. The atmosphere was generally clear while the observations were made, but there was some smoke present most of the time from both the lamp itself and the power unit.

The results[12] for three searchlamps are given in Figs. 2 and 3. All

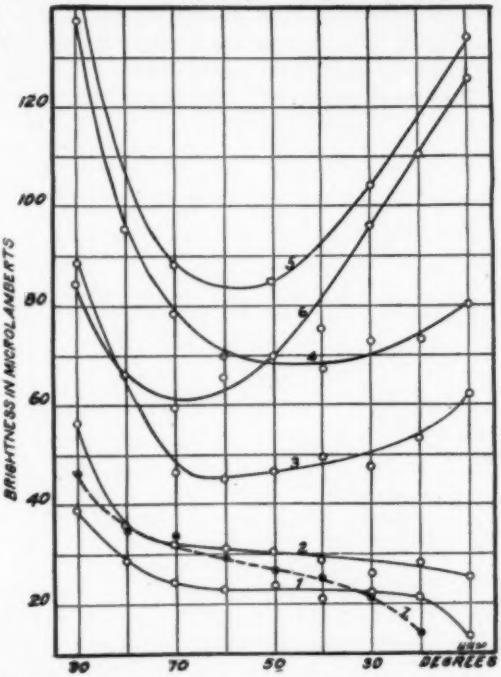

FIG. 2. *Distribution of brightness along the path of the searchlight beams from Sperry Searchlamps.*

Curve 1.—24 in., 75 Amp., vertical; observer at 106.5 ft.
Curve 2.—24 in., 75 Amp., horizontal; observer at 106.5 ft.
Curve 3.—36 in., 150 Amp., vertical; observer at 93 ft.
Curve 4.—36 in., 150 Amp., horizontal; observer at 93 ft.
Curve 5.—36 in., 150 Amp., horizontal; observer at 50 ft.
Curve 6.—60 in., 150 Amp., horizontal; observer at 93 ft.
Curve 7.—24 in., 75 Amp., horizontal; observer at 125 ft.
(Curve 7 is from data of Dec. 4, 1918; all others Nov. 29, 1918.)

[12] Most of these observations were made on Nov. 29, 1918, at the Tenleytown Searchlight Range, near the Bureau of Standards with the cooperation of the Searchlight Investigation Section, U. S. Army. One set, Fig. 2, was made on Dec. 4, 1918. A few observations for the 24 inch Sperry Searchlamp were made July 11, 1919 at the Bureau of Standards.

the curves show a minimum in brightness. From theoretical considerations this minimum may be expected to be in a direction perpendicular to the searchlight beam. The observations were for the most part taken in the field, where the positions were rather inconvenient. In the perpendicular position the photometer was directed at the beam immediately outside the front glass of the lamp, where the presence of smoke and fumes from the arc would augment the brightness; curve 2, Fig. 3 is to be compared with curves 1 and 2, Fig. 2. The initial decrease with angle in curve 2, Fig. 3 is considerably less

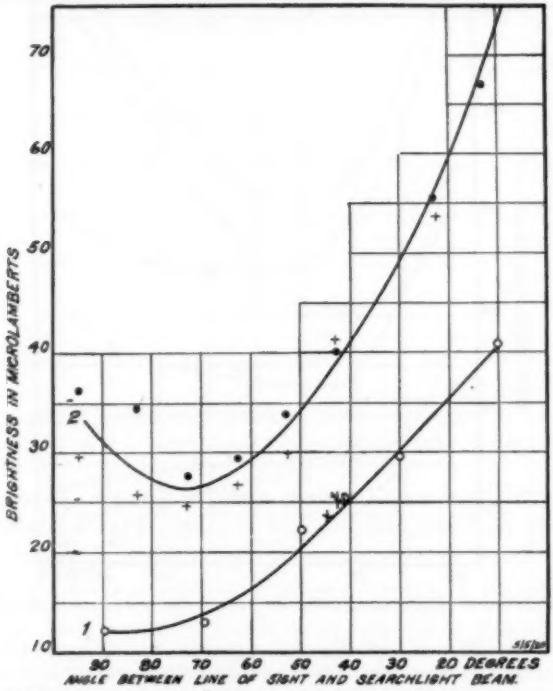

FIG. 3. *Distribution of brightness along the path of the searchlight beams.*
1. Sperry searchlamp 36 in., 150 amp., horizontal; observed at 15 ft., (Nov. 29, 1918).
2. Sperry searchlamp 24 in., 75 amp., horizontal; observed at 12 ft., (July 11, 1919).

than that in curves 1 and 2, Fig. 2. There is, however, a very abrupt rise in brightness similar to that found in case of the 36 and 60 inch searchlamps. The form of the curves is affected chiefly by two conditions; the focussing of the beam and the distance from the lamp at which the observations were made. Curves 4 and 5, Fig. 2, and curve

1, Fig. 3, for the 36 inch searchlamp show the effect of distance, being in these cases 93, 50 and 15 feet respectively. Curves 4 and 6, Fig. 2, illustrate the effect of the focal length of the mirror upon the brightness of path of the beam. In these instances the mirrors subtended equal angles at the arc, but had focal lengths of 14 inches (35.5 cm) and 25 inches (63.5 cm) respectively. Under certain conditions the size of the luminous field surrounding the target has great influence upon the smallest area perceptible.[13]

It is of interest to note that the brightness of the path of the beam horizontal is somewhat greater than that of the beam vertical.

POLARIZATION IN THE DIFFUSED LIGHT

To determine the per cent of polarized light diffused from the beam, a Martens[14] polarization photometer was employed as described by him. The photometer *M* Fig. 4 was set so that its optic axis was

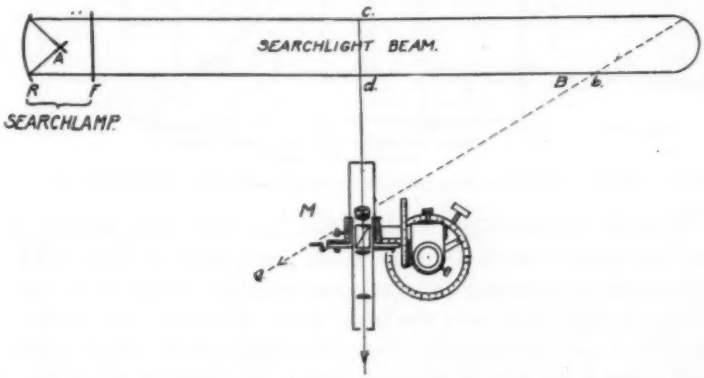

FIG. 4. *Disposition of apparatus for determining the extent of polarization along the path of the beam.*

F, front glass; A, arc; R, reflector; B, angle between line of sight and beam; M, Marten's polarization photometer.

perpendicular to the beam in the following manner. By means of a transit the photometer was first placed at some point on a line *cd* perpendicular to the beam passing through the transit; then to orient its optic axis, the photometer was brought to bear on a lamp placed

[13] This was observed in experiments on the Contrast Sensibility of the Eye (B. S. Sci. Pap. No. 366). Very recently the work of Martin (A study of the Performance of Night Glasses, Bul. No. 3, Dept. of Sci. & Ind. Res., London) who reports some very definite data upon this point has come to our notice.

[14] Phys. Zeit. *1*, pp. 229–303; 1900.

at the transit. These settings could be more accurately done when the beam was horizontal than when it was vertical.

Three sets of observations when the beam was approximately vertical are given in Fig. 5. The atmosphere traversed by the beam in these instances was directly over the part of the Bureau grounds where automobiles had been parked during the day so that the dust content had probably been enhanced.[15]

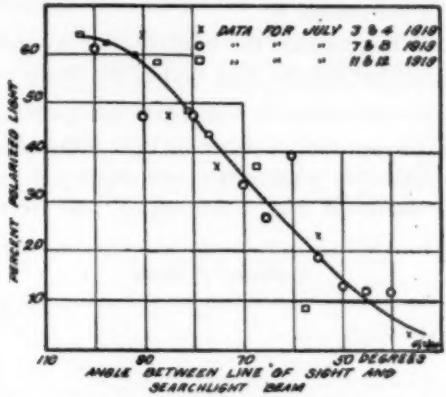

FIG. 5. *Amount of polarization in searchlight beam when directed vertically.*

In some measurements made[16] when the beam was directed south and elevated at about 45° over woods and against a south wind, the percentage of polarized light was considerably higher than that obtained at other times with the beam either vertical or horizontal. The data show that the direction where the maximum amount of polarized light exists is not exactly perpendicular to the direction of the flux, but at a larger angle. The average size[17] of the particles may therefore be in the neighborhood of 140 to 160 mμ (160×10^{-9}m). This shift in the direction of maximum polarization was also noticed in several cases when the beam was horizontal, but the direction of maximum is usually not so well defined.

Measurements with the beam in the horizontal position are given in Fig. 6. In the first two sets (July 6-7, July 7-8) the beam was over dry ground at a height of 12 feet. During the third, (July 11-12) the

[15] Consult report of the Relative Spectral Transmission of the Atmosphere, B. S. Sci. Papers, No. 398; 1920, where observations on the transmission over this space suggest this.

[16] July 2 and 3, 1919.

[17] See Burton, Physical Properties of Colloidal Solutions, p. 114.

ground was moist from rains of the previous day. The per cent of polarization is considerably less than one would expect in diffusion from "scattering" by molecules alone or by very small particles. There seems to be only a slight asymmetry in the directions with the

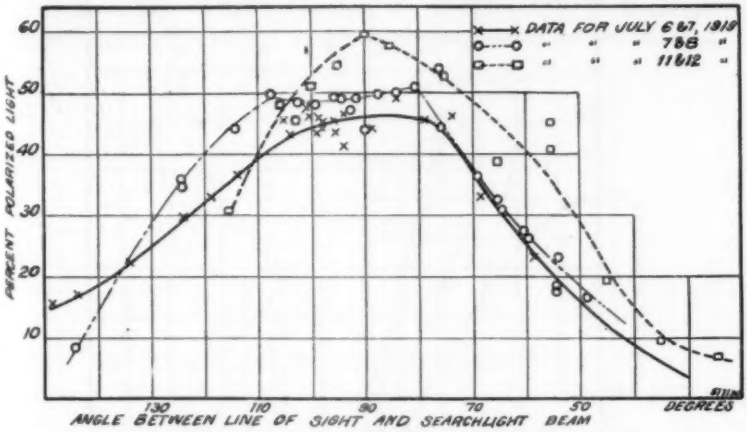

FIG. 6. *Amount of polarization in searchlight beams when directed horizontally.*

flux and against the flux. Both facts suggest the presence of reflection by larger masses. This inference is also borne out by the ill defined maximum in the curves.

COMPARISON OF OBSERVATIONS WITH THEORY

In Fig. 7 some curves of Figs. 2 and 3 are re-drawn in a manner that permits a direct comparison with the distribution (full line curve) that one would expect from the simple theory of scattering by small particles. From theory the range in relative brightness from the minimum in the perpendicular direction, to the maximum in the direction backwards from the incident beam, can only be from 1 to 2. Actually the range, in case of the 36 inch searchlamp, is at least 1 to 4. The theoretical curve[18] would probably not be reliable for directions approaching that of the beam, because the assumption of a fortuitous distribution of phase of the disturbances emanating from the diffusing points (molecules or particles) will not hold in directions approximating that of the beam of light. In such directions the phase of all disturbances may be related so that interference may take place. No safe inferences

[18] As pointed out by Rayleigh, Scientific Papers, *2*.

can be drawn regarding the correctness of theory in this particular, because of the presence of much smoke and fumes in the immediate

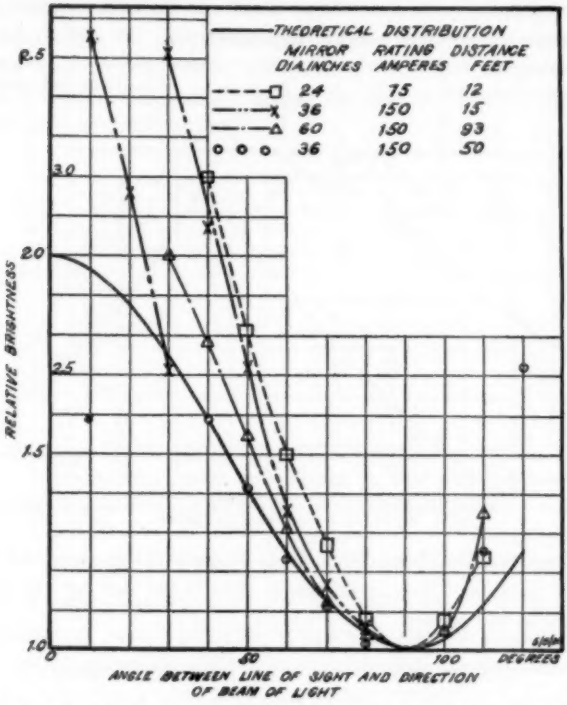

FIG. 7. *Comparison of distribution of brightness along searchlight beams with the theoretical distribution.*

vicinity of the searchlamp. Account must be taken, too, of the fact that in the actual case the diffusing space is a cylinder or cone with axis along the searchlight beam.

IV. PRACTICAL IMPORTANCE OF LIGHT DIFFUSED FROM SEARCHLIGHT BEAMS

EFFECT ON THE VISIBILITY AT VARIOUS DISTANCES FROM THE SEARCHLAMP

As an observer recedes from the lamp to either side, the depth of the luminous atmosphere in the line of sight decreases with consequent decrease in brightness of the background. The contrast between the

target and field is thus increased, and the least perceptible area is decreased.[19]

For the 36 inch searchlamp of 150 amp. the brightness of the path of the beam to an observer who recedes from the lamp to either side, falls off according to curve 1, Fig. 8. The per cent of brightness difference between field and target increases according to curve 2,

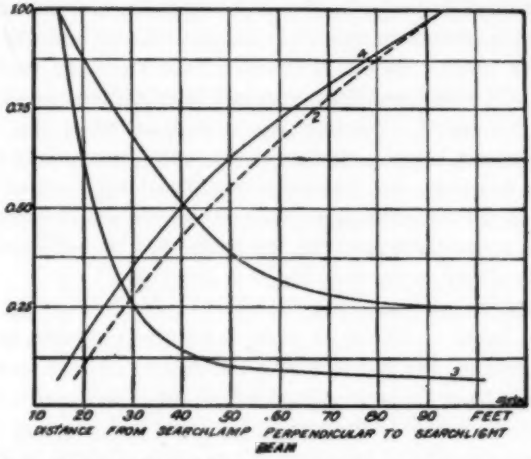

FIG. 8. *The variation of relative visibility of a target when the observer near the lamp is at different distances from the lamp.*

1. *Changes in field brightness with distance.*
2. *Change in contrast of target.*
3. *Change in least visual angle of target.*
4. *The reciprocal of least visual angle as a function of the observer's distance from the searchlamp.*

while the least visual angle[20] necessary for the target to be seen should drop off in accordance with curve 3 somewhat more rapidly than the field brightness. The inverse of the least visual angle is given in curve 4. This latter curve may be compared with that obtained in

[19] See "The Contrast Sensibility of the Eye" already referred to, where relationship is given between brightness of the field and the least visual angle that a target differing in brightness from the surroundings must subtend in order to be visible.

The range in contrast and in the field brightness over which observations were made was very small, but the data are sufficient to enable one to make inference regarding the problem in hand. The conditions were similar to the conditions actually encountered by Searchlight Engineers, even the element of time was allowed to enter as in actual conditions. The latter makes the data somewhat more indefinite than desirable for certain theoretical considerations.

[20] See previous foot-note.

practice by the Army Searchlight Engineers.[21] The Engineers show "the relation between position of observer and his observing ability with relation to targets in air." The curve reported by them is of the same type as our curve 4. It should be noted that the exact nature of this curve depends upon the absolute value of the field brightness as well as upon the contrast, because for a given contrast the target must be larger as the field brightness is decreased but may be smaller for greater contrasts. In the present case both these opposing effects take place. As the observer recedes from the lamp, the surrounding field of diffused light decreases in brightness therefore a larger target is demanded. However, to more than offset this is the fact that the contrast between the target, whose brightness may be assumed to remain constant, and the field of diffused light whose brightness decreases as the observer moves away from the searchlamp, increases, and allows a smaller target. In the range that we have noted a curve of the type shown in curve 4, Fig. 8 is obtained.

In general we may say that the effect of the diffused light from the searchlight beam on vision at night is exactly the same as the effect of fog or mist on the vision during the day.[22] In both cases it is the diffused light from the source illuminating the object that is deleterious. In the one the source is the arc, in the other it is the sun. In case of night vision the effects of diffusion are quite obvious in the clearest atmosphere, whereas in day vision the effects of diffusion by the atmosphere are only plainly obvious in haze, fog, and mist. It is plain therefore that all the considerations which have from time to time been made on the effect of fog and mist upon visibility[23] in daylight may be applied to the visibility of targets under searchlight illumination. Aitken[24] pointed out the effect of fogs and clouds in limiting visibility, and made some qualitative estimates of visibility in different atmospheres. Rayleigh[25] has considered the effect of rain drops on visibility. Trabert[26] has also advanced some excellent considerations as to the effect of raindrops upon vision.

[21] See paper on "Military Searchlights," Trans. I.E.S., *15*, p. 1; 1920.

[22] Of course one must remember that the eye at night is different from the eye during the day for all ordinary use, so that the relative magnitude of effects may greatly differ.

[23] We prefer to use this word always in the present sense and to use "photicity" as a property of the radiation which makes it effective to produce the sensation of brightness (see also Karrer, J. Fr. Inst., p. 79, 1923).

[24] Trans. Roy. Soc., Edinburgh, *20*, p. 76; 1892–93.

[25] Phil. Mag., *47*, p. 475, 1899; or B. S. Sci. Papers, *4*, p. 404.

[26] Meteorol. Zeitsch. *18*, p. 518; 1901.

Several attempts have been made to arrive at a definite method for measuring visibility under various atmospheric conditions. Wiegand[27] suggested a rather arbitrary method. Jones[28] has devised an instrument for measuring the visibility of objects. Such an instrument would be an excellent adjunct in connection with the testing and comparison of searchlamps out of doors where it is very often difficult to interpret results owing to the uncertainty of the effect of the opacity of the atmosphere which changes from day to day and even from hour to hour.[29]

EFFECT ON RANGE OF SEARCHLAMPS

With constant atmospheric conditions the brightness of the path of the beam is a measure of the flux of light along it and with a beam of

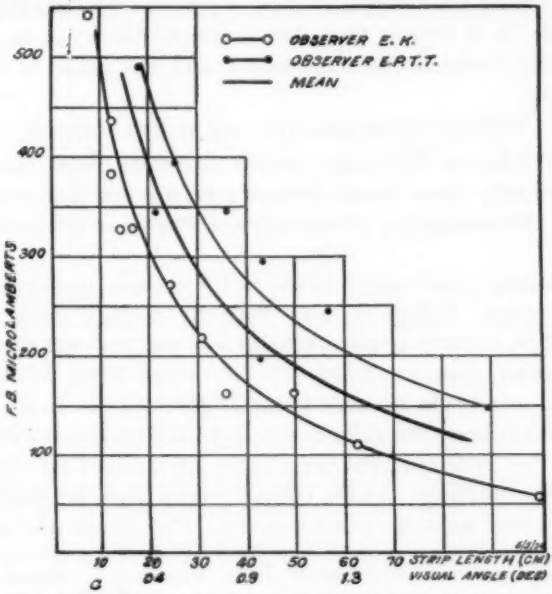

FIG. 9. *The relation between field brightness and size of a target (visual angle) that is 4% brighter than the background.*

constant flux, is an index to the atmospheric conditions. In fact, measurements of the brightness of the path of the beam, would give

[27] Phys. Zeit., *151*, April 1919.

[28] Phil. Mag., *39*, p. 96; 1920.

[29] An instrument of this kind was planned by one of the authors in 1918, while in charge of searchlight testing for the Office of the Chief of Engineers at the Bureau of Standards.

far more direct indications of the effects of the atmosphere in limiting the practical range of searchlights than measurements of the transmission of the atmosphere. Since the brightness of the path of a beam is proportional to the quantity of light lost by diffusion, a very small change in the transmission of a very clear atmosphere may multiply the brightness of the path many times and cause a still greater change in the visibility of the target. It should also be noted that in a non-homogeneous atmosphere the actual magnitude of the effect of the diffusion may also depend upon the distance from the observer of the strata which are diffusing most. For the sake of clearness, curves of Fig. 9 are appended, which show the relationship between the brightness of the background and the minimum target visible, for two observers when the contrast is about 4 per cent.[30] Their mean is also indicated. It is evident then that in general there can be no simple relationship between the transmission and the range of any given searchlamp.

EFFECT OF ELIMINATING POLARIZED PORTION

It is well known that under certain conditions objects may be seen more distinctly when viewed through a nicol prism than when viewed directly. For example, a polarizing piece may be an aid to clear seeing in fogs.[31]

A polarizing prism may also aid in the clearest atmosphere on the brightest days. Foliage is more distinctly outlined through a nicol prism. The relative position of observer and the sun is important in all of these cases.

For the searchlight beam as for other diffusion due to the turbidity of the atmosphere, the polarization is greatest when the diffusion is least, so that little practical benefit may be expected for vision under searchlight conditions. In the following table, data are given to show more definitely what the effect may be. The change in contrast due to the nicol prism is calculated[32] from $1+a'=\frac{1+a}{1-b}$ where a′ is the

[30] These are taken from the report upon the Contrast Sensibility of the Eye, being curves (4) of Figs. 9 and 10 of that report. These data were also used in the previous discussion of the effect of position of observer on visibility.

[31] It is said of Tyndall that he carried a nicol prism about to increase visibility in the dense fogs of London.

[32] This is seen to follow from the definition of a, viz., $T=(1+a)F$, where T and F are the brightness of the target and background respectively. Since bF represents the brightness due to the polarized portion, the same relation with the nicol prism is $T/2=(1+a')\ (F-bF)/2$. Then from these, $(1+a)F=(1+a')\ (1-b)F$.

contrast with nicol prism, a, the contrast without the prism; and b is the percentage of polarized light in the field.

Dist. ft.	*Field Brightness*		% Polarized	*Contrast*		*Least visual angle deg.*	
	W.O.N.	W.N.		W.O.N.	W.N.	W.O.N.	W.N.
10	562	281	0	.048	.048	0.16	0.41
50	238	118.6	30	.048	.050	0.48	1.04
100	125	62.2	50	.048	.052	1.02	1.68

The assumptions made in these calculations are that the target is at a distance of one mile (1.61 km); its brightness is 4.8% more than that of the field; and remains constant[33] for all positions. The data of

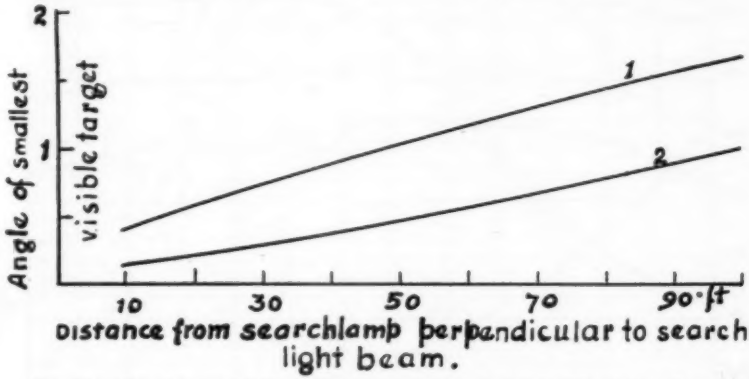

FIG. 10. *The effect on visibility of eliminating the polarized light diffused from the searchlight beam.*

1. Relation between the angle subtended by the least visible target and the observer's distance perpendicularly from the searchlight beam when no nicol prisms are used; 2, same when nicol prism is used.

Target is assumed to be 1 mile from searchlamp; and 4.8% brighter than background for all positions when no nicol is used.

this table are plotted in the curves of Fig. 10. For targets more distant, the elimination of the polarized component in the background would have even greater deleterious effects. However, for an observer very much farther from the searchlamp, where the polarized portion becomes appreciable, the deleterious effect wanes. These data are

[33] See curve 2, Fig. 8 which shows that the brightness increases as one moves away to either side of the searchlamp. Higher brightness levels will also tend to eliminate the difference between vision with and without the nicol prism.

meagre and there are not sufficient at hand to carry these considerations further.

V. THE COLOR OF THE PATH OF SEARCHLIGHT BEAMS

We have no measurements relating to the color of the path of the searchlight beams, but it is of interest to make some theoretical consideration regarding it in order to bring out the relative importance of the various factors involved.

The color of the beam itself depends in the first place upon the color of the light source. In the military high power searchlamps the arc

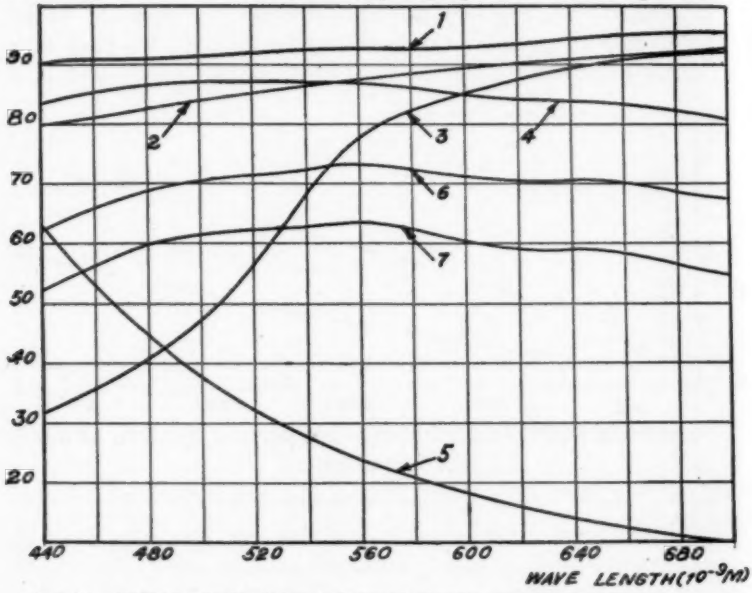

FIG. 11. *Spectral reflection factors (reflectance) of various searchlight mirrors.*

1. *Silver, light incident from air.*
2. *Silver, light incident from glass.*
3. *Gold, light incident from air.*
4. *Transmission factor (transmittance) of the glass of a mirror.*
5. *Relative spectral diffusion of the atmosphere ($\frac{1}{\lambda^4}$ in accord with the simple theory).*
6. *Reflectance of silver glass mirror (2×4).*
7. *Spectral depletion factor of silver glass mirror and glass door of a searchlamp (6×4).*

is used, which is bluish. As secondary factors in the coloring are the mirror and glass door of the searchlamp. These are usually of plate glass with a greenish tint. Their effect is appreciable, however, only

when metals other than silver, such as gold for example, are used. Finally, at considerable distances from the searchlamp the coloring effect of the atmosphere traversed becomes apparent.

The color of the path of the beam will depend upon all of these and in addition will depend upon the way in which the atmosphere diffuses

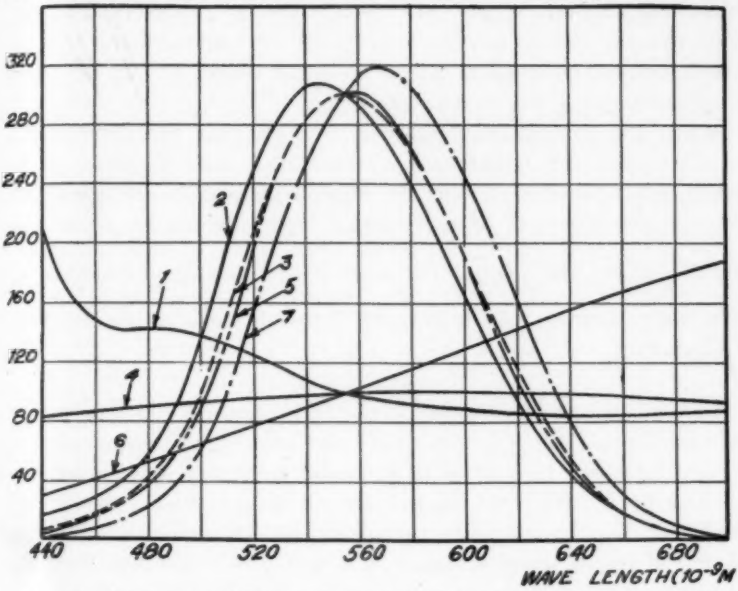

FIG. 12. *Relative spectral intensity (curves 1, 4, 6) and luminosity (curves 3, 5, 7) for various light sources.*

1, 3, high intensity searchlamp arc, 75 amp.;
4, 5, blackbody at 5000 K.
6, 7, blackbody at 3000 K.
2, relative photicity curve.

light of different wave-lengths. To formulate, let the source of light emit at wave-length λ with intensity L_λ; let the searchlamp parts (mirror and front glass) transmit and reflect S_λ; let the atmosphere transmit A_λ and diffuse D_λ at the same wavelength.

The intensity of the diffused light is $L_\lambda\ S_\lambda\ A_\lambda\ D_\lambda$. The magnitude of A_λ will depend upon the distance from the searchlamp.[34] If we consider only points immediately outside of the lamp this factor may

[34] How this factor varies for various wavelengths may be seen in B.S. Sci. Pap., No. 366; 1919 or Bul. Bur. S. p. 377; 1919. Rayleigh (Phil. Mag. *41* p. 107 & 274; 1871) has indicated how the color of a beam varies with distance when diffusion alone is considered.

be dropped (i.e. $A=1$). The factor D_λ may be taken, in accordance with the simple theory of scattering to be $1/\lambda^4$ (plotted in curve 5, Fig. 11). When the searchlamp is equipped with a front glass and a silvered glass mirror, the factor S_λ is made up of at least two factors, the reflection from the silver against glass, (Curve 2, Fig. 11) and the transmission (curve 4, Fig. 11) through glass. The factor resulting from these is plotted in curve 6, Fig. 11. At other times S_λ may simply be the reflection factor of the silver (curve 1, Fig. 11) or of gold (curve 3, Fig. 11) used for the reflector.

When the photometric brightness is of interest the difference in relative photicity[35] of radiation P_λ at various wave-lengths must be taken into account. So that the relative distribution of luminosity through the spectrum of the diffused light is given by a function $\phi(L_\lambda S_\lambda \frac{1}{\lambda^4} P_\lambda)$. The color of the path of the beam is determined by this function, while the brightness is proportional to the integral, $\int \frac{L_\lambda S_\lambda P_\lambda}{\lambda^4}\, d_\lambda$.

In practice the light source is the high intensity, the low or the medium intensity arc. For the first[36] we have actual measurements of L_λ (Fig. 12, curve 1)[37]; for the latter two we have none, but for definiteness we have substituted black bodies of temperatures 3,000 K and 5,000 K,[38] (Fig. 12, curves 4 and 6 respectively). The relative luminosity ($L_\lambda P_\lambda$) of these sources is indicated in curves 3, 5 and 7 respectively of Fig. 12, the relative photicity also being indicated in curve 2.

In Fig. 13 (odd numbered curves) is indicated the relative spectral distributions ($L_\lambda S_\lambda$) of the radiant power in beams from searchlamps equipped in various ways to which the data discussed above are applicable. The even numbered curves of Fig. 13 are derived from these by multiplying ordinates by $\frac{1}{\lambda^4}$, to represent the relative spectral

[35] Trans. I.E.S. p. 522, 1918 or Jour. Amer. Opt. Soc. 1920. These data are for the high levels of adaptation and are not strickly applicable to the light diffused out of the beam. We prefer to use the phrase relative photicity when the effectiveness of radiation for producing the sensation of brilliance is referred to. See foot note 23 above.

[36] B. S. Tech. Pap. 168, 1920.

[37] All curves of this and following figures have ordinates at wavelength (555mμ) equal to unity, where the relative photicity is maximum.

[38] We do not assume that the black bodies are identical with the actual arcs, but give them as illustrative of the ideas. The temperatures (Kelvin) are on the centigrade scale reckoned from absolute zero. Taken from Priest; J. O. S. A. *6*, p. 462, 1920.

distribution of the radiant power diffused from the beams. The relative spectral brightness[39] of the path of the beams of the various searchlamps is given in Fig. 14. A comparison of the curves of Fig. 14

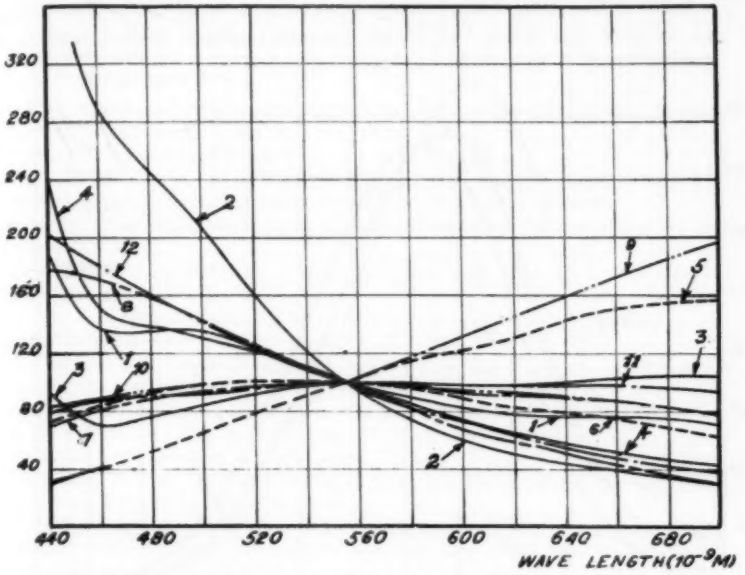

FIG. 13. *Relative spectral power in and diffused from searchlight beams.*

In the beam from a searchlamp equipped 1, with 75 amp. high intensity arc, silver-glass mirror, glass door;

3, with 75 amp. arc, first surface gold mirror;

5, with blackbody radiator at 3000 K, silver-glass mirror and glass door.

7, with black body at 5000 K, silver-glass mirror, glass door.

9, with black body at 3000 K, first surface silver mirror.

11, with black body at 5000 K, first surface-silver mirror.

The relative spectral power diffused from these beams is indicated in curves, 2, 4, 6, 8, 10 and 12 respectively.

with those of Fig. 12 brings out the well known fact that in all cases the light diffused from the beam is relatively bluer than that in the beam.

The yellowing of the beam when a gold mirror is used is shown in curve 2, Fig. 14 (to be compared with curve 1, Fig. 14). The yellower path has of course less brightness and for this reason might be expected

[39] Derived from even numbered curves Fig. 13 by multiplying ordinates by relative photicity P_λ (curve 2, Fig. 12).

to increase the visibility of the target. The difficulty at present is that any such modification of the color of the beam is at a great sacrifice of intensity.[40]

The color of the path of the beam is under given conditions indicative of the color of the beam but itself appears to have little practical

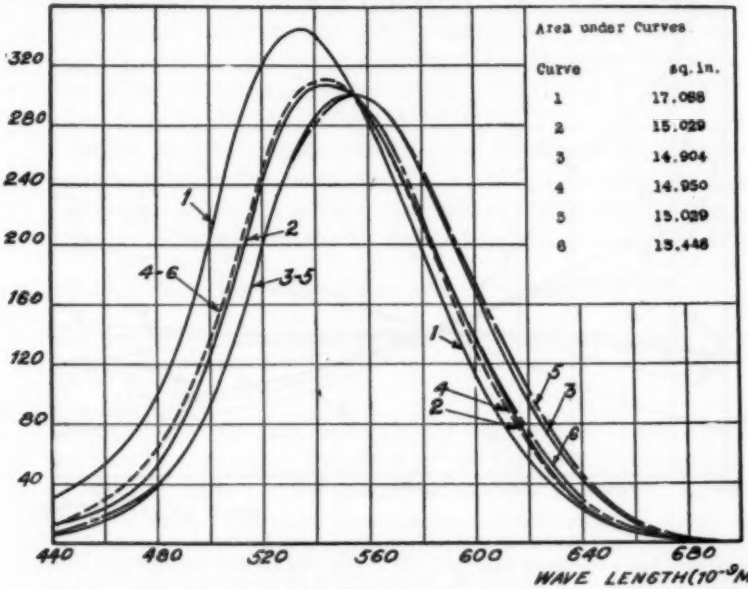

FIG. 14. *Relative spectral brightness of the path of the beams from searchlamps equipped as indicated in the caption of Fig. 13 respectively.*

importance, except in so far as it may also indicate the brightness of the path of the beam.

VI. BIBLIOGRAPHY

We append a list of references on the subject of the diffusion of light and on visibility in so far as it may be related to diffusion. This list includes some that have been cited in the text and others. No claim is made for completeness. We have eliminated all references that may also be found in a Report on Atmospheric Scattering, Sky Polarization, and Allied Phenomena, by Fowle which appeared in the J. O. S. A. *6*, p. 99; 1922, subsequently to the writing of this paper.

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METHODS OF MAKING TEMPERATURE CORRECTIONS IN DIRECT READING POTENTIOMETERS FOR MEASURING HYDROGEN-ION CONCENTRATIONS

By W. T. Bovie

In 1915, (Jr. of Medical Research, *33*, pp. 295-322; 1915) I published an account of a potentiometer for measuring hydrogen-ion concentrations which is direct reading and direct recording and which plots a curve on semi-logarithmic coordinate paper showing the relation between the hydrogen-ion concentration of a solution and any independent variable such as time, or the amount of solution added in the process of titration. I have made a number of models of this potentiometer, some for myself and some for other investigators, and have, since the publication referred to above, made certain improvements in the design which seem to me worthy of record as suggestions for other investigators who may wish to construct similar instruments for their own use.

The direct reading principle, with its multiple logarithmic scale and the semi-logarithmic coordinate paper, have not been changed. These are used in the new design just as described in the paper referred to above, but the wiring diagram of the potentiometer has been changed in order to include a shunt circuit for making temperature corrections. This shunt circuit has been applied to two different potentiometers —one using a quadrant electrometer and the other a galvanometer— as a null instrument. It is the object of this paper to describe this method of temperature correction.

If P is the potential, T the temperature (absolute) and C and C' the concentrations of hydrogen ions bathing the two electrodes, the equation:

$$P = 0.0001983 T^{\circ} \times \log_{10} \frac{C}{C'}$$

which states the relation between the potential and the concentrations of a hydrogen-ion concentration cell may be written:

$$P = 0.0001983 T^{\circ} \times \log_{10} \frac{1}{C'}$$

when the reference electrode is bathed by a normal solution with respect to hydrogen ions.

If a calomel electrode is substituted for the normal hydrogen electrode the difference in potential between the calomel electrode and a normal electrode p must be added (algebraically) to the potential of the hydrogen-ion concentration cell and the equation becomes:

$$P = 0.0001983 T^\circ \times \log_{10} \frac{1}{C'} \pm p$$

If the calomel electrode is made up with a saturated KCl solution and the connection between the calomel electrode and the solution C' is made by a saturated KCl solution, the term p is practically constant for a range of hydrogen-ion concentrations in C' from normal to 1×10^{-12} normal. For this range of concentrations the relation between P and $\log_{10} \frac{1}{C'}$ is linear for any temperature T°.

Rearranging the above equation we see that the relation between the potential of the concentration cell and the logarithm of the hydrogen-ion concentration of the solution bathing the variable electrode is a function of the absolute temperature T°:

$$\frac{1}{T^\circ} P = 0.0001983 \times \log_{10} \frac{1}{C'} \pm p$$

Changes in the value of T° alter the rate at which P changes as $\log_{10} \frac{1}{C'}$ is varied but does not alter the linear relation between P and $\log_{10} \frac{1}{C'}$. The method of temperature correction to be described is based on this fact.

THE DIRECT READING PRINCIPLE

Referring to Fig. 1, a source of emf., e.g., a dry cell A, supplies current to the circuit CDR. The amount of current flowing through the circuit and therefore the potential drop in the portion CD is controlled by the rheostat R. The concentration cell forms a part of the shunt circuit CGH. H is a sliding contact which may be moved along CD and G is the null instrument, such as a galvanometer or a quadrant electrometer, which by its deflections indicates the direction of current flow in the shunt circuit. If the position of the contact H is such that the potential difference between C and H is just equal to and in the same direction as the potential difference between the electrodes of the concentration cell, no current will flow through the shunt circuit and the null instru-

ment will show no deflection. In practice, therefore, one moves the contact *H* until no deflection is shown. The potential drop along *CH* is then equal to the potential of the concentration cell. Now in the direct reading instrument previously described a hydrogen-ion concentration scale is applied to the circuit *CD* and the position of the contact *H* indicates directly on this scale the concentration of hydrogen ions.

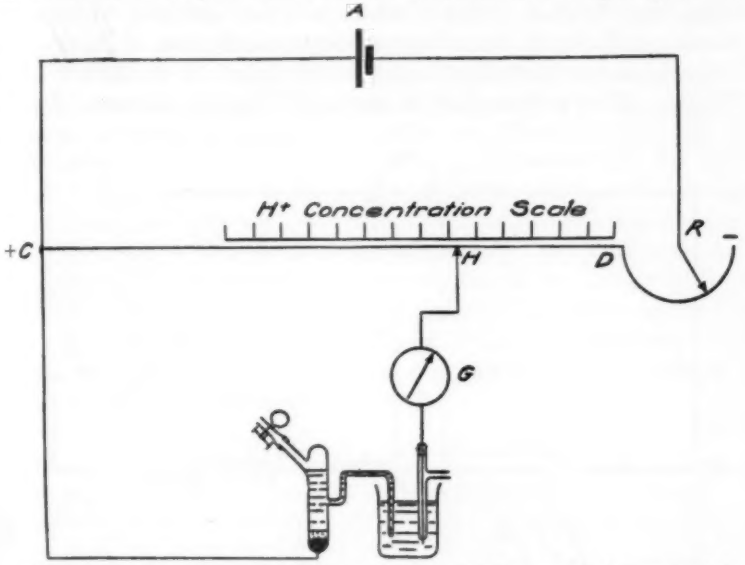

FIG. 1. *A wiring diagram of the direct reading potentiometer method of measuring hydrogen-ion concentrations.*

Since the potential of the concentration cell is a function of the temperature we must either provide a separate hydrogen-ion scale for each temperature at which measurements are to be made or we must provide a means of altering the potential gradient in *CD* in such a manner that the proper voltage range will be included within the limits of the scale. Obviously it is more practical to use the latter method. One method of applying these shunt circuits is given in Fig. 2, and another in Fig. 3.

THE TEMPERATURE CORRECTION

Two dry cells—*A*—Fig. 2, connected in series supply current to the circuit *BDR* and thus maintain a potential gradient in the portion of the circuit between *C* and *D*. Three shunt circuits are associated with

this main circuit, one *BSGF*, includes a standard cell *SC*, another *FGSH*, includes the hydrogen-ion concentration cell, and the third circuit, *CI*, includes a rheostat *T*.

The resistance in the circuit between *B* and *F* measured in ohms is one hundred times the emf. of the standard cell in volts. The resistance between *F* and *C* is 23 ohms and between *C* and *D* is 112.9 ohms. The resistance between *C* and *D* should be linear and for a hydrogen-ion concentration scale 28 cm long, should be made about 30 cm in length. The resistance between *C* and *I* is 395 ohms, 75 of which is in the rheostat *T*. The rheostat *R* should have a resistance of about 100 ohms.

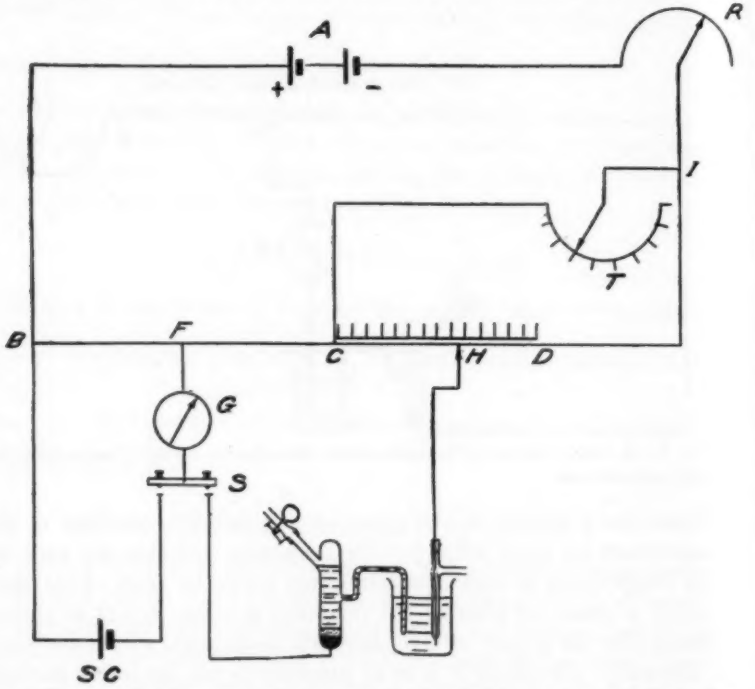

FIG. 2. *A wiring diagram of the direct reading potentiometer method of measuring hydrogen-ion concentrations, using a galvanometer as null instrument.*

Now if when current is flowing through the circuit *BDR* the switch *S* is closed so as to include the galvanometer into the shunt circuit *BSCF* and the rheostat *R* adjusted until the galvanometer shows no deflection, then the drop in potential between *B* and *F* will be just equal

to the emf. of the standard cell and one hundredth of an ampere of current will be flowing through the circuit *BDR*.

As pointed out above, if the instrument is to read hydrogen-ion concentrations directly, it is necessary that the proper potential gradient be maintained between *C* and *D* and this potential gradient is a function of the temperature at which the hydrogen-ion concentrations are measured. The potential gradient in the circuit between *C* and *D* is controlled by the rheostat *T* in the shunt circuit *CI*. It is convenient to fix a pointer and temperature scale to this rheostat in order that the proper adjustment may be readily made. It will be found expedient to use a potentiometer for calibrating this scale. One proceeds as follows: Electrical contacts are fastened to the circuit *CD* with a distance between them equal to the length of the hydrogen-ion concentration scale (28 cm in this instrument). These contacts are connected with the calibrating potentiometer and while maintaining a current of one hundredth of an ampere in the circuit *BDR*, the rheostat *T* is adjusted so that the potential difference between the contacts which have been applied to *C-D* has the values given in the following table, and the corresponding positions of the pointer are marked on the temperature scale.

TABLE OF POTENTIAL DIFFERENCES

Temperature	Volts	Temperature	Volts
15°	0.7994	28°	0.8358
16	.8022	29	.8386
17	.8050	30	.84098
18	.8078	31	.84378
19	.8106	32	.84658
20	.8134	33	.84924
21	.8162	34	.85204
22	.8190	35	.85484
23	.8218	36	.8575
24	.8246	37	.8603
25	.8274	38	.86296
26	.8302	39	.8659
27	.8330	40	.8687

A further correction is necessary, namely a correction for the influence of temperature upon the potential difference between the calomel electrode and a normal hydrogen electrode. Moreover, unless the calomel electrode is formed out of materials having a high degree of purity, the difference in potential between it and the normal hydrogen

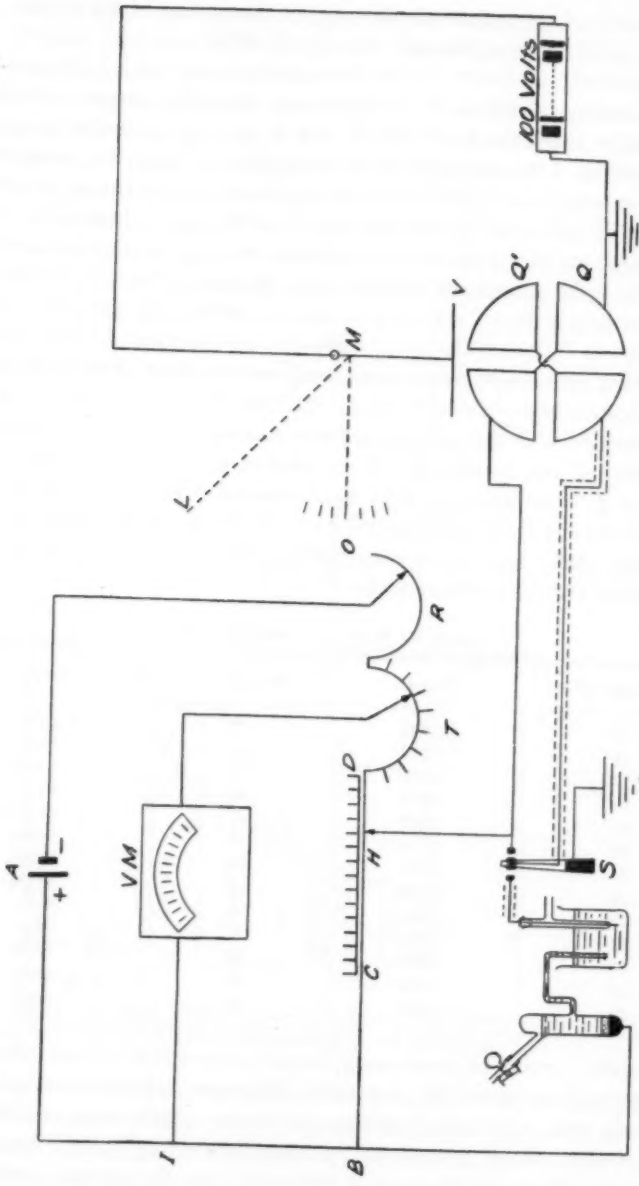

FIG. 3. *A wiring diagram of the direct reading potentiometer method of measuring hydrogen-ion concentrations, using a quadrant electrometer as null instrument.*

electrode will differ from the theoretical value by several millivolts. Both the temperature correction and the correction due to impurity of materials are additive and may, therefore, be made by simply sliding the entire hydrogen-ion concentration scale along the resistance *C-D*. It is for this reason that *C-D* is made a little longer than the hydrogen-ion scale. The proper position of the hydrogen-ion scale is found as follows: After the rheostat *T* has been set to the temperature at which the measurements are to be made and the rheostat *R* has been adjusted so that 0.01 ampere of current is flowing through the circuit *BDR*, a solution of known hydrogen-ion concentration such as a buffer solution is placed in the concentration cell, and the contact *H* is adjusted until the galvanometer shows no deflection when the concentration cell is closed into the circuit *FGSH* by means of the switch *S*. The hydrogen-ion scale is then moved along *C-D* until the pointer on the contact *H* indicates the known hydrogen-ion concentration of the solution. Because of the electrical stability of the calomel electrode the position of the hydrogen-ion concentration scale need not be changed as long as there is no change in the temperature at which the hydrogen-ion determinations are made.

In the instrument shown in Fig. 3, the dry cell *A* is the source or current for the circuit *BDTR*. The shunt *I-T*, includes a voltmeter *VM*, and the shunt *BSH* includes the concentration cell and a quadrant electrometer. The total resistance of the circuit *BDTR* should be large enough so that not over 0.01 ampere of current is taken from the dry cell. The distribution of resistance in the various parts of the circuit *B-R* is determined by the resistance in the portion of the circuit *C-D* which subtends the hydrogen-ion concentration scale. The amount of resistance in the various parts of a particular instrument are given below. Resistances for any other instrument can be calculated from these values by simple proportion. The scale of this particular instrument is 28 cm long and the resistance of 28 cm of the circuit lying between *C* and *D*, which is linear, is 104.3 ohms. The resistance of *B-C* is 29 ohms. The total resistance between *C* and *D* is 114 ohms. The resistance of the rheostat *T* is 16 ohms, and of the rheostat *R* is 77 ohms. The total resistance of the circuit *BCDTR* is 235 ohms. The distance between *C* and *D* is 31 cm.

A temperature scale is fixed to the rheostat *T* in such a manner that an index attached to the sliding contact moves over it. The total resistance of the circuit between *B* and the temperature graduations on this scale are given in Table 2.

TABLE 2

Temperature	Ohms	Temperature	Ohms
15°	156.57	28°	149.75
16	156.02	29	149.25
17	155.48	30	148.83
18	154.94	31	148.33
19	154.40	32	147.84
20	153.87	33	147.38
21	153.34	34	146.89
22	152.82	35	146.41
23	152.30	36	145.96
24	151.78	37	145.48
25	151.27	38	145.04
26	150.76	39	144.54
27	150.25	40	144.08

The voltmeter *VM* has a range of 1.2 volts. When the index of the rheostat *T* is set to the temperature reading on the scale at which the hydrogen-ion concentration measurements are to be made, and the rheostat *R* is adjusted until the voltmeter reads at 1.2 volts, then the potential gradient in the circuit *C-D* is correct, and if the hydrogen-ion concentration scale is adjusted to the proper position the instrument will read hydrogen-ion concentrations directly. The correct position of the scale is determined by means of a solution of known hydrogen-ion concentration as described above on page 1241.

It will be observed that with this arrangement the voltmeter is always read at one of the graduations on its scale. Settings closer than 0.01 volt can readily be made. Further, the graduation at the end of the voltmeter scale is used as a determinate point by the makers in graduating the scale and it is usually accurate. It is possible, therefore, to use an inexpensive voltmeter.

The connections of the quadrant electrometer are clearly indicated in the figure. *Q'* is the variable pair of quadrants, *Q* the fixed pair. The variable quadrants are connected to the switch *S*, which is formed of insulating material such as Bakelite or hard rubber, but has a handle of metal. The metal handle is grounded at *E'*. The connection between the hydrogen electrode and the switch and between the switch and the variable quadrants should be shielded with a metallic conductor which should also be connected with the ground (not indicated in the drawing). For these shielded circuits, single strand armoured cable such as is often used in the wiring of automobiles will be found most convenient. The fixed quadrants are grounded at *E*.

The charge on the vane V is maintained by a one hundred volt battery—one terminal of which is grounded. Movements of the vane V are indicated on the screen C by a beam of light L, which is reflected by the mirror M, fastened to the staff of the vane suspension.

In operation, the switch S is turned so that both pairs of quadrants are grounded. The four quadrants are all at the same potential and the position of the vane V as indicated by the spot of light on the screen 0, is noted as the "zero position." When the switch S is turned so as to connect the variable quadrants with the hydrogen electrode the current flows until the potential difference between the quadrants and the point B is equal to the electromotive force of the concentration cell. The contact H is adjusted until the potential difference between H and B is equal to the potential difference between Q' and B. All four quadrants are again at the same potential and the spot of light on the scale will again be in the "zero position."

When a quadrant electrometer is used as a "null" instrument in the manner here described the hydrogen-ion concentration cell is always on a practically open circuit and the only current flowing through the cell is that which flows into or out of the variable quadrants Q'. The amount of current required to change the charge on the variable quadrants is of course negligible. A larger, but also negligible amount of current may flow from the vane to the variable quadrants, due to leakage across the air gap between them.

This leakage is ordinarily very small. It is not large enough to be significant except when the instrument is used in a field of ionizing radiation such as might exist in the neighborhood of an x-ray plant or radioactive substances.

A quadrant electrometer has certain advantages over a galvanometer as a null instrument because of the negligible amount of current taken from the concentration cell. The area of the hydrogen electrode, for example, may, when experimental conditions demand it, be made very small—a mere pin point will suffice. Again, because the concentration cell is always on open circuit, detrimental current cannot be taken from it through the potentiometer,—no matter how far from the correct reading the contact H is when the cell is closed into the shunt circuit by means of the switch S. One may, therefore, set the contact H at the hydrogen-ion concentration of the equivalent point in an electrometrical titration and add acid or alkali as the case may be, until the spot of light on the scale 0, is at the "zero position." Thus the spot of light may be used precisely as we use an indicator—titrating it

backwards and forwards over the equivalent point as often as desired. When used in this manner the potentiometer serves as a hydrogen concentration indicator, the shifting of the spot of light marking any departure from the desired concentration between normal acid and normal base so that titrations may be made with even greater rapidity than when the usual color indicator methods are used. The determinations, moreover, have a degree of precision equal to, and usually superior to the most elaborate color indicator methods.

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THE MEASUREMENT OF IONIZATION CURRENTS

By Roy Kegerreis

The universality of the use of Roentgen rays and the recent rise to prominence of their therapeutic use has caused much work to be done along the broad lines of intensity measurements. It is a new field and presents many difficulties, since it is usually necessary that the work be done at the place where the Roentgen apparatus is installed rather than in the measuring laboratory.

This type of investigative work is generally carried out by the biophysics department, which is a recent outgrowth in most medical research organizations. Considerable controversy is going on among physicists concerning the biologic equivalence of ionization measurements. Ionization is not in general directly proportional to biological effect. It is also a fact that ionization measurements are the only practical method of metering the dosage of these scientific entities.

It is with the hope of assisting the large and important group of workers who have to do with the measuring of ionization currents that the following methods of induction protection and leakage control are offered.

Many things encountered in scientific investigations are measurable only by indirect methods. The Roentgen ray is one of these; the measurement is generally based on the ionization of gases which is an accompanying phenomenon. It has been known for a long time that a gas which is practically nonconductive electrically becomes conductive when ionized, and that, if a sufficient voltage be applied, the conductivity is proportional to the ionization, which in turn is proportional to the intensity of the rays; but the extreme minuteness of ionization currents is not generally appreciated.

Under the usual conditions of experiment, with small ionization chambers, the ionization currents have about the same magnitude as would result if the current through an ordinary fifty-watt incandescent lamp were divided into a million parts, and then one of these parts subdivided into ten thousand parts. It may readily be seen that external influences of seemingly trifling magnitude may produce disturbances of marked importance.

The principle to be described had its inception in the consideration that the measurement of ionization currents due to Roentgen rays,

radium or any other phenomenon, where only exceedingly small currents are encountered, is difficult and complicated because of leakage, which is uncertain and more or less a matter of conjecture. Errors due to leakage, induction and infiltration of currents, not directly due to the phenomena involved, affect the indication of the measuring instrument, and must be determined separately and then added or subtracted in order to arrive at a more reliable measurement. The separate determination is often exceedingly difficult, and at times impossible. It is preferable to eliminate such spurious currents, so far as possible, rather than correct for them.

The solution of the problem from this angle is based on the conception that these leakages may be entirely eliminated by governing the voltage distribution around the conductors of a circuit, so that its insulation problems are less exacting. The insulation is divided into two stages, and a conductor which is charged to a certain voltage is inserted between the two stages of insulation and prevents the tendency for leakage across one of the insulating stages. This is in large part an elaboration of the guard ring principle.

It may be said here that the very old textbooks on electrical measurements devote much more space to this exceedingly desirable principle than do the newer volumes. The increasing complication of our modern practices may be the cause of this. We now have, besides ordinary direct currents, sinusoidal alternating currents of twenty-five, sixty, 500 and up to and above 100,000 cycles per second. The voice waves of speech and the modulated waves encountered in radio engineering with their higher harmonics make a vast realm which relegates the fundamental phenomenon of direct currents to a very minor position in the modern text.

Electrostatic and electromagnetic induction play very important parts in all disturbances encountered in measurements in which the pulsating high voltages of Roentgen ray production are at hand.

The physical laws of electrical influence between wires carrying currents are well known. Two parallel wires with currents in the same direction attract each other, and repel each other with currents in the opposite directions. Even if one of the wires carried no current, similar, as well as more involved, phenomena are prevalent due to electromagnetic, as well as electrostatic induction. Most, if not all, of these associated phenomena assert themselves to a very pronounced extent when the values of the currents and the potentials which cause them change. It is these effects which are so pronounced and annoying

in Roentgen ray work. Workers in this field have perhaps observed that it is possible, with a screw driver held in the hand, to draw a visible spark from a nail driven in a wall. Here are two conductors, one, approximately six inches long, and the other two inches long, with inductive sparks playing between them, and it is necessary to do experiments in such an atmosphere with refinements seldom needed in other practices. All possible means for the elimination of inductive interference must be resorted to.

The wires and tubular conductors which are used, act as antennae, either to transmit or to receive. The induced currents which spring up in the circuits of the measuring apparatus vitiate the results in most peculiar fashions and constant precaution is necessary to eliminate their influences.

It may be noted for instance, that a rectified current (*A* Fig. 1) which consists of regular pulses induces electromagnetically a corresponding pulse at the beginning and at the end of each single pulse (*B* Fig. 1). This means that a rectified 60 cycle alternating current, which is used in the ordinary Roentgen ray installation, induces 240 pulses each second in all the surrounding conductors. Conditions are aggra-

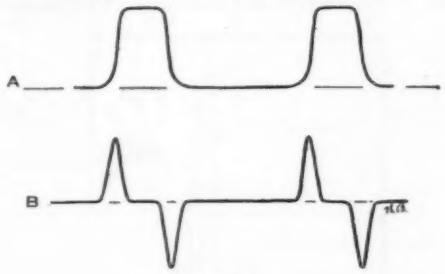

FIG. 1

vated by the fact that the slopes at the beginning and at the end of the pulses in the rectified high voltage direct current are very steep; which is another way of stating the instantaneous variation of current with respect to time is very large at the beginning and end of each of the more or less rectangular-shaped pulses. The value of the induced voltage during these very short periods of increase and decrease will be considerable since induced voltage is proportional to di/dt, as expressed in the notation of the calculus.

In most actual installations the conditions are far more complicated than the above described conditions since the makes and breaks of the

rectifier are not gradual increases or decreases, but consist of oscillating surges which have extreme frequencies and are backed up by considerable power.

Electroscopes and electrometers are ordinarily used for measuring these very small currents. Galvanometers may be used if the current is somewhat greater. Another type of instrument which may sometimes be used, and which should receive greater and more considerate attention is a sensitive ammeter, known commercially as a "micro-ammeter." The rating of these instruments is not given on the same basis as that which is used when stating the sensitivity of a galvanometer. The current cited for expressing the sensitivity of a galvanometer is that which is required to give a deflection of one millimeter at the end of an optical pointer one meter long, whereas the range of a micro-ammeter is commonly expressed as the current required to produce full scale deflection; this is entirely different. For example, a micro-ammeter of two micro-amperes range, with 100 scale divisions, would compare favorably with a galvanometer of a sensitivity equal to 2 by 10^{-8} amperes.

With the wiring diagram of Fig. 2, all leakage, induced currents, and currents due to infiltration are avoided. This is accomplished by

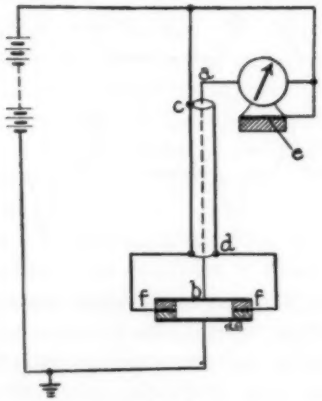

FIG. 2

surrounding the conductor *ab*, which connects the meter and ionization chamber, with a conducting sheath *cd*, while the insulated conductor *e* underneath the meter prevents current leaking away from the internal mechanism itself. There remains, however, the leakage through the insulation within the ionization chamber itself.

It is also perfectly feasible entirely to eliminate this troublesome defect if the insulation between the two parts of the ionization chamber is divided in two stages, and a conductor *ff* inserted between them. There will be no leakage with connections as shown in Fig. 2. The insertion of the conductors *ff* between the two plates of the ionization chamber redirects the leakage; it may even increase it, but its passage through the meter circuit is entirely obviated. Leakage of itself causes little trouble; the objectionable feature of it lies in the fact that it is usually confused with other currents.

Ionization measurements are comparable only if the voltage impressed on the ionization chamber is the same in all the work or if conditions of absolute saturation exist. A convenient way to provide for testing the voltage is to wire suitable resistances as shown in Fig. 3. Resistances *g* and *h* are in series across the battery, and act as potentiometer resistances. When the double key is pressed, the meter

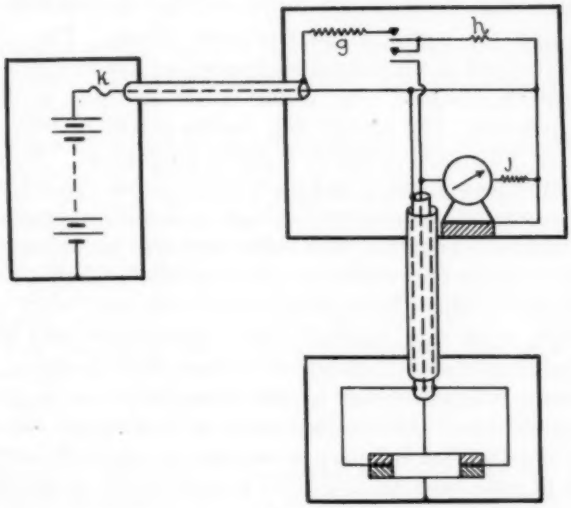

FIG. 3

circuit has the voltage across resistance *h* impressed on it; the total voltage is accordingly measured because of the invariable relation between these fixed resistances. For one particular case the resistances are as follows:

$g = 100{,}000$ ohms
$h = 100.77$ ohms
j and meter resistance $= 15{,}000$ ohms

Then 10 microamperes flow through the meter, when the double key is pressed, if 150 volts are impressed on the circuit.

There is only one cable connecting the meter to the ionization chamber. The battery itself may be placed at a remote position and connected to the meter by means of a two-conductor, concentric cable.

Many of the phenomena encountered depend on the capacity between the various conductors. Very long leads may introduce sufficient capacity in the meter circuit to make it respond slowly to changes in the thing being measured. Mechanical rigidity is especially important since the swaying or disturbance of any conductor may so alter its capacity, or generate frictional electricity to such a degree, that all current flow will be stopped or even momentarily reversed. Any length of cable may be used without disturbing influences, because of the shielding and the method of connections employed. This means, for instance, that the indicating instrument may be placed in an office remote from the roentgen ray treatment rooms. The cable may be definitely fixed to the ionization chamber, or a three-conductor jack and receptacle provided. All wiring should be made as direct and short as possible. The surrounding sheaths should be very carefully constructed with good electrical contacts throughout. It is well to solder flexible leads to all hinged doors or covers, and such connections in multiple are well worth while. Closed or semiclosed loops are to be avoided whenever possible. Well soldered ground connections to some one large cross-section conductor are especially desirable. Straight, direct wiring is always better than coiled conductors, which in reality are nothing more than solenoids, and consequently may have considerable inductive drop of potential between their terminals.

A voltage, sufficiently high for saturation, must be applied to an ionization chamber. The voltage required depends on the size and shape of the chamber and may be supplied by dry cells, storage batteries, or in some cases preferably by a small direct current generator giving constant voltage.

These principles may, in part, be applied when an electroscope or electrometer is being used. It is common, when using variable voltage instruments of this type, to make the observations over only a small range of voltage, as for example, between 80 and 70 per cent of the total applied voltage (discharge assumed). If the intermediate conductor were kept constantly charged to 75 per cent of the voltage, the leakage would be practically eliminated.

This scheme of leakage prevention may be beneficially applied to the measurement of the current through a roentgen ray tube. It is, however, advisable that the insulation between the two concentric conductors be made very large.

Those who desire to employ this method will inquire for "three-conductor concentric cable." This is not at present a regular product of commerce, but may be made up in short lengths by taking small-sized, single-conductor, armored cable, such as is used for automobile headlights, insulating it by hand, and then drawing it into metallic gas hose, or the like.

It is unfortunate that all these suggestions require increased complications in setting up the apparatus. The benefits will come, however, during the taking of direct observations, as will be evident to those who have had long experience with leakage corrections.

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Colour Vision.—A Discussion of the Leading Phenomena and their Physical Laws. By W. Peddie (London: Edward Arnold and Co., 1922, 208 pages, $4.00).

In this review the reviewer takes the position that his task is merely to advise the reader in definite terms of the nature and scope of the book, without taking issue to any marked extent with the author on controversial questions involved in theories of color vision.

In the present case this information can, in large part, be conveyed most cogently by advising the reader forthwith as to what this book is *not*. The full title of a work such as that given above naturally calls forth a certain expectation as to its scope and content. This book certainly does not fulfill the promise of its title. The prospective reader who expects to find either an exposition of the present status of knowledge of the experimental facts of "color vision" or a discussion of the theories which have been advanced to comprehend them, will be greatly disappointed.

This failure of the book to cover the field indicated by its title is perhaps best illustrated by a list of authors and subjects which do not appear in its index. These lists, to a certain extent, exemplify only the defective nature of the index, but a perusal of the book indicates that the text itself is defective to nearly the same extent. To cite only a few examples, the following authors of papers bearing directly on the presumed subject matter do not appear in the index:

Dalton, Grassmann, Seebeck, Holmgren, Donders, Wundt, Nagel, Rayleigh, Trendelenberg, Lummer, Köllner, Grant Allen, Frank Allen, Nutting, Hyde, L. A. Jones, Ladd-Franklin, Troland, Blanchard, Reeves, Ferree, Rand, Piéron, Westphal, Bidwell, Crova, Ebbinghaus, Greenwood, Sherrington, Whitman.

The following are a few significant omissions of subjects in the index:

Scotopia, Photopia, Deuteranope, Protanope, Induction, Photometry, Heterochromatic photometry, Spectrophotometer, Colorimetry, Colorimeter, Frequency, Critical frequency, Wavelength, Macula, Macular pigmentation, Santonin, Rayleigh equation, Zone theory, Visibility, Hue, Chroma, Visual acuity, Brightness.

In spite of the author's expressed admiration for the genius of Maxwell (pp. 3-5), the text gives hardly more of Maxwell than the mention of his name and laudation of his genius; there is no account of Maxwell's actual contributions to the subject.

Somewhat more attention is given to Young; but the reader in search of a definite account of Young's formulation of his famous theory will be much better served by Mayer's essay on "The History of Young's Discovery of his Theory of Colors" (Am. Jour. of Sci. *109*, pp. 251-267; 1875).

As for more recent work, some mention is made of Ives, Abney, Steindler, Watson, Houston, and others, but the treatment is entirely inadequate to meet the needs of anyone searching for information on the contributions of these workers to the subject. Although some consideration of it is tacitly implied in several passages, (e.g., pages 147, 157, 193 *et seq.*) the only formal mention of the Duplicity Theory is that given in the following paragraph (p. 204):—

Nevertheless, the view that the rods are efficient in perception of luminosity only while the cones are responsive to colour stimulus, which is the basis of the Duplicity Theory of v. Kries, is not antagonistic to the trichromatic theory of Young and Helmholtz. It merely limits the mode of application of the latter by raising the question of structure.

There is no bibliography and, except for perhaps half a dozen incomplete citations, the book is entirely devoid of references to the literature.

These lists of omissions might be extended, but these few random illustrations will be sufficient to indicate to any active worker in this subject the limitations of the book. Aside from giving an extended treatment of the three-components theory, it does not, in even a partial way, fill the long felt need of such workers for a modern treatise on the subject, although its title aroused in the reviewer the hope for such a book. The reader with similar needs is advised to turn again to Parsons' "Introduction to the Study of Colour Vision" (Cambridge, 1915) supplemented by Troland's admirable monograph "The Present Status of Visual Science," (Washington, National Research Council, 1922).

Having disillusioned the reader of his natural expectations, it remains to indicate briefly the actual content of the book and the style of presentation. It may best be described as a eulogistic essay on the Young-Helmholtz theory. The basis of treatment is Part II of the second edition of Helmholtz' Handbuch der Physiologischen Optik. Much of the text is simply an English translation of parts of this work and in many other sections, where direct translation is not resorted to, the treatment given by Helmholtz is followed very closely.

There is also some extension and development of the theory which is apparently original. In addition, a certain amount of post-Helmholtzian experimental work of Steindler, Exner, Ives, Abney, Watson, McDougall, and others is adduced as evidence in support of the theory, but the discussion of this work is rather superficial. For the most part the "theory" is presented simply as the formal relations which subsist among visual phenomena on the hypothesis that the visual sensation is a function of three variables. No specific anatomical or physiclogical hypothesis is insisted upon. Indeed, the question of the "mechanism" is explicitly waived:

> Even the postulate of three sets of nerve fibres was only tentative. Given the formal laws as determined by the strict processes of physical science, the trichomatic theory seeks only to determine the formal consequences which are then to be compared with the results of observation and experiment. The determination of the mechanism is a matter for the anatomist and the physiologist alone to investigate, . . . (p. 204).

Criticisms based on the opinion that the physiological implications of the theory are inadmissible are thus forestalled, but the question arises as to whether or not any "theory" remains. It would appear to some that what the author has left is merely a statement of empiric laws, inviting the invention of a "theory" to "explain" them. Scant attention is given to other theories and little critical attention to many facts which have seemed to others to render the Young-Helmholtz theory inadequate.

The subject matter consists almost entirely of generalized theory. The treatment is largely mathematical. There is little reference to experimental work while the actual content of experimental data is almost negligible.

The style is that of a rather impassioned plea for the Helmholtz theory, and the present reviewer leaves the book with a distinct impression that the author has striven to make his point, somewhat after the manner of political orators, by laudation of the genius of Newton, Young, Maxwell, and Helmholtz rather than by a purely judicial consideration of experimental evidence. The parts of Helmholtz most dwelt upon are those which are omitted entirely in the third edition (1911) and represent a point of view which seems to be regarded as *passé* by most modern authorities. (Cf. Nagel, pp. VII, VIII of the Preface of third edition of Helmholtz' Physiol. Optik, I.) The present reviewer regards it as unseasonable as well as beyond the reasonable limits of a review to attempt any extensive criticism of the position taken by the author. (A review doing this to a certain extent has already appeared while the present review was being prepared, Nature, June 16, 1923, pp. 799-800. For impartial discussions of the various color theories and their difficulties, the reader is referred to:—(1) von Kries in third edition, Helmholtz' Physiol. Optik. vol. 2, pp. 354-378; (2) Parsons' "Introduction to the Study of Colour Vision," pp. 193-299; (3) Troland's "Enigma of Color Vision," American Jour. of Physiol. Optics, Oct., 1920, and Jan., 1921.)

A few specific comments may be made.

(1) On pages 16-17, the following passage occurs:

> Ordinary daylight is called "white" light. Its composition, as made evident by Newton's method of prismatic resolution, is subject to slight variations according as the day is clouded or unclouded or according as it is observed in the morning or at mid-day or in the evening, and so on. Under otherwise the same conditions it depends on the amount of blue sky which is visible, and on the general colour tone of the landscape, but these variations are of no special account for general observational purposes.

This is supported (p. 18) by a quotation from Helmholtz. In view of the experimental fact that the quality of various phases of daylight covers a range as great as from candle-light (direct light from the horizon sun) to blue sky, this statement seems somewhat too strong. Helmholtz himself (Physiol. Optik, 2nd Ed., p. 338) characterized the difference between lamplight and daylight as "ausserordentlich gross."

(2) The author lays great stress upon the importance of simplicity in hypotheses, (pp. 7, 9, 205, and elsewhere). No one would gainsay the desirability of simple hypotheses, from a didactic point of view; but unfortunately it does not seem likely that any simple hypothesis will ever adequately account for all of the phenomena of color vision. (Cf. Troland: Am. Jour. Physiol. Optics, 1, p. 321; Oct., 1920, and "The Present Status of Visual Science", p. 52.)

(3) In Fig. 1 and the discussion relating to it (p. 45) an error occurring in the second edition of Helmholtz (p. 402) is copied without question. The sentence which reads: "The former represents the effect with blue light, the latter with reddish yellow light" should read: "The former represents the effect with reddish yellow light (wave-lengths 670, 605, 575 mμ), the latter with blue light (wave-lengths 505, 470, 430 mμ)." The correct statement is given in Koenig's Gesammelte Abhandlungen zur Physiologischen Optik, pp. 128-129.

(4) In discussing the color triangle diagram (p. 55) we find the following:

> The usually visible colours are those included within the boundary formed by the spectrum curve and the straight line joining its extremities. This line is the locus of the purples, ranging from red to violet, which form the only natural colours not included in the range of the spectrum.

It would be more accurate to give the locus of the purples as an area instead of as a line.

(5) On page 76 another error is copied from Helmholtz (2nd Ed., p. 462) in spite of the fact that in Helmholtz, it is corrected in the errata, p. 1008. Moreover, the error in question apparently has a vital bearing on the author's argument.

(6) Fig. 3, p. 54, copied from Helmholtz (Physiol. Optics, 2nd Ed., p. 340) is defective in that the symbol "V" which should appear at the lower right corner is missing although reference is made to it in the text (p. 65).

This book is very difficult reading. This is of course due in a large part to the inherent complexity of the subject, but a confused and

inadequate nomenclature and lack of clarity in expression add materially to the reader's difficulties. Although this book is not unique in this respect, still it appears to be a step backward rather than forward in the matter of nomenclature. A few examples of confused nomenclature and more or less ambiguous or confusing statements may be cited:

(1) Throughout the book "colour" is used indiscriminately as a stimulus term and as a sensation term, with apparently no explicit definition.

(2) On page 11, the three attributes of "colour quality" are given as "intensity," "colour tone" and "degree of saturation." On page 135 they are given as "whiteness," "colour," and "luminosity." On the same page "colour value" is used in a sense apparently synonymous with the established meaning of "hue" and with the author's own terms "colour" and "colour tone." On page 138, the author speaks of a "common whiteness or saturation." On page 140 he uses "purity" apparently as a synonym for "saturation."

(3) The section heading "Variations in Sensitiveness to Colours" (p. 13) conveys no meaning to the reviewer, while it certainly indicates a confusion of ideas as to stimulus and sensation.

(4) The section heading "Visible Colours" (p. 55) would apparently imply the existence of "Invisible Colours." The division of colors into two classes, namely, "visible" and "invisible" does not appear to be a valuable concept.

(5) We find the following terms used in a very confusing manner with a notable lack of explicit definitions and without any noticeable attempt to lead the reader into clear and definite concepts:

"ground colours" (pp. 36, 71), "fundamental colours" (pp. 9, 11, 51, 55, 70, 120), "fundamentals" (pp. 3, 55), "fundamental perceptions" (p. 56), "fundamentally independent perceptions" (p. 56), "absolute fundamentals" (pp. 133, 151), "derived fundamentals," (p. 121), "true absolute fundamentals" (p. 143), "standard spectrum fundamentals" (p. 137), "fundamental sensations" (pp. 52, 129), "fundamental positive sensations" (p. 150) "fundamental stimuli (pp. 135, 136, 141), "fundamental excitations" (p. 92), "elementary excitations" (pp. 28, 88), "colour sensations" (p. 118), "simple sensations," (p. 30), "components of sensation" (pp. 25, 151), "component sensations" (p. 25), "sensation components" (p. 154), "external stimulus" (p. 148), "effective external stimulus" (p. 136), "subjective stimulus" (p. 142), "subjectively active stimuli" (p. 142), "colour value" (pp. 28, 135, 144), "colour sensation value" (p. 141), "true colour sensation value" (p. 143), "sensation values" (pp. 135, 141), "sensation curves" (pp. 118, 137, 152), "colour perceptions" (p. 14), "pure colour sensation" (p. 137), "colour fraction" (p. 144), "colour impression" (p. 11), "light impression" (p. 12).

(6) In his translation of Helmholtz the author has generally rendered "Empfindung" as "perception" (pp. 25, 26, and elsewhere) apparently not recognizing the distinction now commonly made by psychologists between "sensation" and "perception," a distinction which indeed was made by Helmholtz himself between "Empfindung" and "Wahrnehmung."

(7) On page 17, the following statement occurs:

If all kinds of light which are present in the incident white light are absorbed in practically the same proportions as those in which they are present in the incident light, the scattered

light, although it is feebler than the incident light, nevertheless has the same composition; and the body is said to be a white body.

This can hardly be regarded as an exemplar of clearness of concept nor of perspicuity.

(8) On page 43, Fechner's law is stated thus:

The general statement then would be that the *change in* sensation is proportional to the logarithm of the *change of* intensity, or that the *change in* sensation varies in arithmetical progression as the *change in* intensity varies in geometrical progression.

The reviewer has italicized words which he would suggest be omitted from the definition. (Cf., e.g., Külpe, "Outlines of Psychology" Eng. Trans. by Titchener, pp. 164-165, and Dunlap, "Elements of Scientific Psychology" p. 127.)

(9) On page 51, it is stated—

That any color if suitably darkened by the addition of black or diluted by the addition of white can be matched by a mixture of three suitably chosen fundamental colours.

(10) On page 145 the following passage occurs. The comments in brackets are interpolated by the reviewer.

"The extreme peripheral parts of the retina are nearly or entirely blind to all colour. The central part of the retina, where cones lie exclusively or in preponderance, possesses in normal eyes full trichromatic power. It is to be expected therefore, that the field of vision for white light exceeds that for coloured light; and this is the case." [Does this plain statement mean that homogeneous radiant energy would evoke no sensation when incident on the peripheral parts of the retina?] "The region immediately within the monochromatic belt exhibits yellow-blue dichromasy." [Why call the region which is "blind to all colour" a "*monochromatic* belt?"]

Certainly one *sine qua non* for the production of a satisfactory treatise on color is the formulation and adoption of a rational and intelligible nomenclature, and the primary feature of such a nomenclature must be a sharp distinction between stimulus terms and sensation terms. [Cf. Troland: (1) "Report of Colorimetry Committee," J. O. S. A. & R. S. I., Aug.; 1922; and (2) "The Present Status of Visual Science."]

As providing for ready reference, a unified and extensively developed treatment of the Young-Helmholtz Theory in its most generalized form, this book will serve a useful purpose in spite of its defects. It is notable for its formal mathematical treatment of features of the theory which have been omitted by other commentators. It should be particularly valuable to students whose knowledge of German is not sufficient to allow of the free reading of Helmholtz in the original. Such readers will, however, find it advantageous to regard it as an *aid* to the study of the original rather than as a substitute for it.

IRWIN G. PRIEST

INDEX TO VOLUME 7

A

Abbot, C. G.: Apparatus and Results of Solar Radiation Work, 61.
Abbreviations adopted by this Journal, 500.
Aberration in Aspherical Optical Systems (Irwin Roman), 861.
Abney Effect, Further Studies of (L. T. Troland and C. H. Langford), 70.
Absorption and Diffusion by the Atmosphere on Signal Lights and Projectors (Enoch Karrer), 943.
Absorption Bands, Infrared (H. M. Randall), 45.
Absorption of HCl in Infrared (B. J. Spence and C. Holley), 169.
Acuteness of Vision of School Children, Tests for (J. M. McCallie), 70.
Adams, Elliot Q.: A Theory of Color Vision, 66.
Adams, Elliot Q.: A Comparison of the Fechner and Munsell Scales of Luminous Sensation Value, 70.
Aircraft, The Determination of the Altitude of (W. C. Brombacher), 719.
Allen Frank: On Reflex Visual Sensations, 61, 583.
Allen, Frank: On Reflex Visual Sensations and Color Contrast, 913.
Altitude Indicator, An Optical (John A. C. Warner), 225.
Altitude of Aircraft, The Determination of the (W. G. Brombacher), 719.
Ammeter-Voltmeter, A Projection (Howard M. Elsey), 403.
Apparatus for Admitting Gas Into a Vacuum (J. B. Johnson and H. W. Weinhart), 175.
Apparatus, Exhibit of Scientific, 677.
Apparatus, Notes and News, 1067.
Apparatus, Standardization of (J. M. Roberts), 973.
Arcs, Note on Brauer's Theory of the Function of the Cathode (A. Trowbridge and K. T. Compton), 321.
Aspherical Aberration (Irwin Roman), 861.
Astigmatism of Oblique Bundles in Case of a Single Lens, Abolition of (James P. C. Southall), 219.
Atoms, Chemical and Spectroscopic Properties of Excited (K. T. Compton), 955.
Atoms and Electrons, Some Properties of (Frederic Palmer, Jr.), 873.
Auditory Acuity, New Procedure in Tests of (C. C. Bunch), 1161.

B

Barr, C. E. and W. T. Bovie: An Automatic Focusing Device for Photomicrography with Ultraviolet Light or Light of any Wave Length, 1167.
Bazzoni, C B., R. W. Duncan and W. S. Mathews: An Instrument for the Measurement of Photographic Densities, 1003.
Becker, H. G.: A Constant Pressure Blowpipe, 907.
Becker, J. A. and F. W. Warburton: The Use of a Sector Disk in X-Ray Measurements, 127.
Behr, Leo: Multiple Range Potentiometers, 665.
Benford, Frank: A Hemispherical Photometric Integrator, 76.
Benford, Frank: Reflection and Transmission of Parallel Plates, 1017.
Berry, Eugene M.: Diffuse Reflection of Light from a Matt Surface, 627.
Berry, Eugene M.: The Reflection of Light from a Sphere, 679.
Bifilar Pendulum (H. S. Uhler), 263.
Blowpipe, A Constant Pressure (H. G. Becker), 907.
Bovie, W. T.: A Spark Gap With Air-Cooled Electrodes, 1027.
Bovie, W. T.: Methods of Making Temperature Corrections in Direct Reading Potentiometers for Measuring Hydrogen-ion Concentrations, 1235.
Bovie, W. T. and C. E. Barr: An Automatic Focusing Device for Photo-micrography with Ultraviolet Light or Light of any Wave Length, 1167.
Bovie, W. T., and E. L. Chaffee: Electrical Response of the Retina to Stimulation by Light, 61.
Bovie, W. T.; Alice Hampson, and E. L. Chaffee: The Electrical Response of the Retina Under Stimulation by Light, 1.
Brewster's Law (A. Sommerfeld), 501.
Brombacher, W. G.: The Determination of the Altitude of Aircraft, 719.
Bunch, C. C.: New Procedure in Tests of Auditory Acuity, 1161.
Burette, A Direct Reading (Albert M. Jackson), 491.
Burette and Filter Stand (Clifford D. Carpenter), 671.
Burns, Keivin: On the Measurement of Standard Wave Lengths, 419.

C

Carpenter, Clifford D.: An Improved Burette and Filter Stand, 671.
Castleman, Jr., R. A.: A Device for Logarithmic and Semi-Logarithmic Plotting and Finding Powers and Exponentials, 377.
Cathode in Electric Arcs, Note on Brauer's Theory of the Function of the (A. Trowbridge and K. T. Compton), 321.

CHAFFEE, E. L., W. T. BOVIE, and ALICE HAMPSON: The Electrical Response of the Retina Under Stimulation by Light, 1.
CHAFFEE, E. L. and W. T. BOVIE: Electrical Response of the Retina to Stimulation by Light, 61.
CLARK, GEORGE L. and WILLIAM DUANE: A New Method of Crystal Analysis and the Reflection of Characteristic X-Rays, 455.
CLARK, JANET HOWELL: A Photoelectric Theory of Color Vision, 66.
COBLENTZ, W. W.: Thermocouple Measurements of Stellar and Planetary Radiation, 61.
COBLENTZ, W. W.: Some Observations on the Transformation of Thermal Radiant Energy Into Electric Current in Molybdenite, 63.
COBLENTZ, W. W.: Methods and Apparatus in Spectroradiometry, 439.
Color Contrast and Reflex Visual Sensations (Frank Allen), 913.
Color Match and Spectral Distribution (W. E. Forsythe), 1115.
Color Match Photometer for Illuminants (Herbert E. Ives), 75, 243.
Color Measurements from One System to Another, Graphical Aids to the Transformation of (Herbert E. Ives), 75.
Color Measurements of Illuminants by Trichromatic and Monochromatic Analysis (Herbert E. Ives), 75, 287.
Color Mixing and Comparing Apparatus (Hermann Kellner), 77.
Color-Mixture (M. Luckiesh), 657.
Color-Sensation, the Fundamental Facts of (Christine Ladd-Franklin), 66.
Color-Sense, Individual Variations of the Normal (R. H. Sinden), 1123.
Color Vision, A Photoelectric Theory of (Janet Howell Clark), 66.
Color Vision, A Theory of (Elliot Q. Adams), 66.
Color of Daylight at Washington (Irwin G. Priest), 78.
Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion (Irwin G. Priest), 75, 1175.
Colorimetric Standards, Geometric Progression in Optically Prepared (Arthur P. Harrison), 375.
Complementaries, Spectral and the Subjective Saturation of the Spectrum (R. H. Sinden), 69.
COMPTON, K T.: Chemical and Spectroscopic Properties of Excited Atoms and Reversible Effects of Electron Impacts in Gases, 955.
COMPTON, K. T. and A. TROWBRIDGE: Note on Brauer's Theory of the Function of the Cathode in Electric Arcs, 321.
Control of Temperature and Other Physical Quantities, An Improved Method for (Paul E. Klopsteg and W. H. Stannard) 179.
Corrections, 58, 414, 1068.
COTTRELL, CASPER L. and IRWIN G. PRIEST: The Effect of Various Conditions Upon the Determination of the Normal Stimulus of Gray, 73.
Crystal Analysis (George L. Clark and William Duane), 455.
Crystal Plates in Convergent Polarized Light (F. E. Wright), 779.
CURTIS, HARVEY L.: Note on the Shrinkage of Photographic Films, 79, 275.
Curve Fitting and Least Squares (H. S. Uhler), 1043.

D

Densitometer for the Measurement of High Photographic Densities (Loyd A. Jones), 80, 231.
Densitometer (George R. Harrison), 999.
Densitometer (See Photographic Densities).
Diffraction Effects in Optical Pyrometer (C. O. Fairchild and W. H. Hoover), 543.
Diffuse Reflection (Eugene M. Berry), 627.
Diffusion and Absorption by the Atmosphere on Signal Lights and Projectors (Enoch Karrer), 943.
Diffusion of Light from a Searchlight Beam (Enoch Karrer and U. M. Smith), 1211.
Dioptrometer, An Apparatus for Measuring the Power of Commercial Lenses (Henry F. Kurtz), 64, 103.
Disc, A Variable Aperture Rotating Sectored (Herbert E. Ives), 76, 683.
Disk, Photometric Sectored (Enoch Karrer), 893.
Dispersion of a Constant Deviation Spectroscope, A Method of Increasing the (J. F. T. Young), 405.
DUANE, WILLIAM and GEORGE L. CLARK: A New Method of Crystal Analysis and the Reflection of Characteristic X-Rays, 455.
DUNCAN, R. W., W. S. MATHEWS and C. B. BAZZONI: An Instrument for the Measurement of Photographic Densities, 1003.

E

Electron Impacts in Gases (K. T. Compton), 955.
Electroscope for Measurements in Radioactivity, A Projection (Howard M. Elsey), 385.
ELSEY, HOWARD M.: A Projection Electroscope for Measurements in Radioactivity, 385.
ELSEY, HOWARD M.: A Projection Ammeter-Voltmeter, 403.
Emfs, A Precise Method of Measuring Small (T. R. Harrison and Paul D. Foote), 389.
Exhibit of Optical Instruments, 82.
Exhibit of Scientific Apparatus, 677.

Exposure, on the Relation Between Time and Intensity in Photographic (Loyd A. Jones and Emery Huse), 80.
Extensometer Amplifier (S. R. Williams), 1011.

F

FAIRCHILD, C. O. and W. H. HOOVER: Disappearance of the Filament and Diffraction Effects in Improved Forms of an Optical Pyrometer, 77, 543.
FARWELL, H. W.: Some Laboratory Notes, 901.
Fechner and Munsell Scales of Luminous Sensation (Elliot Q. Adams), 70.
FERREE, C. E.: The Theory of Flicker Photometry, 71.
FOOTE, PAUL D. and T. R. HARRISON: A Precise Method of Measuring Small Emfs and The Change in Thermoelectric Power of Tin at the Melting Point, 389.
FOOTE, PAUL D., A. E. RUARK, C. C. KIESS, and F. L. MOHLER: The Excitation of Spark Spectra of Rubidium and Caesium in a Low Voltage Arc, 62.
FOOTE, PAUL D.: A. E. RUARK and F. L. MOHLER: The D_2 Zeeman Pattern for Resonance Radiation, 415.
FORSYTHE, W. E.: Color Match and Spectral Distribution, 1115.
FOULK, C. W.: A New Form of Precision Hydrometer, 327.
FREHAFER, M. KATHERINE: New Tables and Graphs for Facilitating the Computations of Spectral Energy Distribution by Planck's Formula, 74.

G

Gas Admitting Apparatus (J. B. Johnson and H. W. Weinhart), 175.
GIBSON, K. S.: Direct-Reading Photoelectric Measurement of Spectral Transmission, 693.
GIBSON, K. S., and E. P. TYNDALL: The Visibility of Radiant Energy, 68.
Grating Spectrograph, Vacuum (Theodore Lyman), 495.
Gray, The Effect of Various Conditions Upon the Determination of the Normal Stimulus of (Irwin G. Priest and Casper L. Cottrell), 73.
Gray Light, Progress on the Determination of Normal (Irwin G. Priest), 72.
GREEN, J. B.: Note on Lloyd's Mirror, 299.

H

Halos, Some Unusual (W. J. Humphreys), 62.
HAMPSON, ALICE, E. L. CHAFFEE, and W. T. BOVIE: The Electrical Response of the Retina Under Stimulation by Light, 1.
HARRINGTON, E. L.: A Low Voltage, Low Wattage Mercury Lamp, 689.
HARRISON, ARTHUR P.: Geometric Progression in Optically Prepared Calorimetric Standards, 375.
HARRISON, GEORGE R.: A Simple Densitometer for Accurate Work, 999.
HARRISON, T. R. and PAUL D. FOOTE: A Precise Method of Measuring Small Emfs and The Change in Thermoelectric Power of Tin at the Melting Point, 389.
HAY, DONALD L.: Modified McLeod Gauge, 1015.
Helical Spring, Coupled Oscillations of (A. Sommerfeld), 529.
Helium Atom, Model of the (A. Sommerfeld), 509.
HERSCHEL, WINSLOW H.: The Improved MacMichael Viscosimeter, 335.
HOBBS, HERMANN E. and HERBERT H. KIMBALL: A New Form of Thermoelectric Recording Pyrheliometer, 707.
HOLLEY, C. and B. J. SPENCE: The Infrared Absorption of Hydrogen Chloride in the Region 3.5μ and at 200°K, 169.
HOOVER, W. H. and C. O. FAIRCHILD: Disappearance of the Filament and Diffraction Effects in Improved Forms of An Optical Pyrometer, 77, 543.
Horopter, Cyclophoria and Apparent Vertical Meridian (Charles Sheard), 81.
Hues, Subjective Saturation of Spectral (R. H. Sinden), 1123.
HULBURT, E. O.: A Simple Prism Spectrograph, 205.
HUMPHREYS, W. J.: The Optics of the Atmosphere, 61.
HUMPHREYS, W. J.: Some Unusual Halos, 62.
HUSE, EMERY, and LOYD A. JONES: On the Relation Between Time and Intensity in Photographic Exposure, 80, 1079.
Hydrometer, A New Form of Precision, (C. W. Foulk), 327.

I

Illumination, On the Relation of Intrinsic Brightness to (James E. Ives), 78.
Infrared Absorption Bands (H. M. Randall), 45.
Infrared Absorption of HCl (B. J. Spence and C. Holley), 169.
Infrared Spectrum of Long Wave Lengths, Summary of Investigations in (W. Weniger), 517.
Instruments, Life Testing of Mechanical (Frederick J. Schlink), 1031.
Integrator, A Hemispherical Photometric (Frank Benford), 76.
Integrating Spheres (A. H. Taylor), 76.
Intensity Measurements of Spectral Lines (John K. Robertson), 983. (See Densitometer.)
Interference Figures, (F. E. Wright), 779.

Interferometer, The Hilger Microscope (F. Twyman), 635.
Ionization Currents, The Measurement of (Roy Kegerreis), 1245.
IVES, HERBERT E.: Graphical Aids to the Transformation of Color Measurements from one System to Another, 75.
IVES, HERBERT E.: A Color Match Photometer for Illuminants, 75, 243.
IVES, HERBERT E.: Simultaneous Color Measurements of Illuminants by Trichromatic and Monochromatic Analysis, 75, 287.
IVES, HERBERT E.: A Variable Aperture Rotating Sectored Disc, 76, 683.
IVES, HERBERT E.: A Chart of the Flicker Photometer, 363.
IVES, JAMES E.: On the Relation of Intrinsic Brightness to Illumination, 78.

J

JACKSON, ALBERT M.: A Direct Reading Burette, 491.
JOHNSON, J. B. and H. W. WEINHART: Apparatus for Admitting Gas Into a Vacuum, 175.
JONES, LOYD A.: An Instrument (Densitometer) for the Measurement of High Photographic Densities, 80, 231.
JONES, LOYD A.: A Special Sensitometer for the Study of the Photographic Reciprocity Law, 80, 305.
JONES, LOYD A. and EMERY HUSE: On the Relation Between Time and Intensity in Photographic Exposure, 80, 1079.
JONES, LOYD A. and A. L. SCHOEN: Preliminary Note on the Spectral Energy Sensitivity of Photographic Materials, 81, 213.
JONES, L. T.: Another Mercury Vapor Pump, 537.

K

KARRER, ENOCH and A. PORITSKY: An Improved Slide Wire Resistance, 277.
KARRER, ENOCH: A Photometric Sectored Disk, Variable and Electrically Readable While in Rotation, 893.
KARRER, ENOCH: The Effect of the Diffusion and Absorption by the Atmosphere on Signal Lights and Projectors, 943.
KARRER, ENOCH and U. M. SMITH: Diffusion of Light from a Searchlight Beam, 1211.
KEGERREIS, ROY: The Measurement of Ionization Currents, 1245.
KELLNER, HERMANN: Color Mixing and Comparing Apparatus, 77.
KELLNER, HERMANN: An Apparatus for Testing the Perception of Depth, 857.
KELLNER, HERMANN: A New Ocular Micrometer, 889.
KIESS, C. C., F. L. MOHLER, PAUL D. FOOTE, and A. E. RUARK: The Excitation of Spark Spectra of Rubidium and Caesium in a Low Voltage Arc, 62.
KIMBALL, HERBERT H. and HERMANN E. HOBBS: A New Form of Thermoelectric Recording Pyrheliometer, 707.
KIRKPATRICK, PAUL: A Continuously Variable Rheostat with Constant Contacts, 195.
KIRKPATRICK, PAUL: A Device for Rotating Spectrometer Crystals, 539.
KLOPSTEG, PAUL E. and W. H. STANNARD: An Improved Method for the Control of Temperature and Other Physical Quantities, 179.
Kundt's Tube, Design of a (Otto Stuhlman, Jr.), 905.
KURTZ, HENRY F.: The Dioptrometer, An Apparatus for Measuring the Power of the Commercial Lenses, 64, 103.

L

Laboratory Apparatus, Standardization and Limitation of, Types and Sizes of (J. M. Roberts), 973.
Laboratory Notes (H. W. Farwell), 901.
LADD-FRANKLIN, CHRISTINE: The Fundamental Facts of Color Sensation, Being the Minimal Requirements of a Color Sensation Theory, with Illustrations in Color, 66.
Lamp, A Low Voltage, Low Wattage Mercury (E. L. Harrington), 689.
Lamps, The Design of Large Incandescent (P. G. Nutting), 399.
LANGFORD, C. H. and L. T. TROLAND: Further Studies of the Abney Effect, 70.
LAPP, C. J.: A Simple Device for Recording Sound Waves, 661.
Lens, Abolition of Astigmatism of Oblique Bundles in Case of a Single (James P. C. Southall), 219.
Lens Construction, The Application of Parabolic Surfaces in, (W. B. Rayton), 65, 197.
Lenses, An Apparatus for Measuring the Power of (Henry F. Kurtz), 103.
Lenses, Grinding Non-Spherical (Robert V. Morse), 705.
Lens System, Methods for Machine Calculation of Rays Through A (C. W. Woodworth), 673.
Lens Systems, Aspherical (L. Silberstein), 82.
Life Testing of Mechanical Instruments (Frederick J. Schlink), 1031.
Light, the Electrical Response of the Retina Under Stimulation by (E. L. Chaffee, W. T. Bovie, and Alice Hampson), 1.
Lloyd's Mirror, (J. B. Green), 299.
Logarithmic and Semi-Logarithmic Plotting, A Device for (R. A. Castleman, Jr.), 377.
LUCKIESH, M.: Demonstrating Color-Mixture, 657.

Luminosity Ratios of Spectral Complementaries (R. H. Sinden), 69.
Luminous Sensation Value, Comparison of the Fechner & Munsell Scales of (Elliot Q. Adams), 70.
LYMAN, THEODORE: The Vacuum Grating Spectrograph, 495.

M

MAST, S. O.: Photic Orientation in Organisms, 61.
MATHEWS, W. S., C B. BAZZONI, and R. W. DUNCAN: An Instrument for the Measurements of Photographic Densities, 1003.
MCCALLIE, J. M.: A Critical Study of the Snellen Letters and the "Illiterate" E Tests for the Acuteness of Vision of School Children, and a Proposed Substitute for These Tests, 70.
MCKEEHAN, L. W.: Extraordinary Diffraction of X-Rays, 63.
McLeod Gauge (Donald L. Hay), 1015.
Measurement of Standard Wave Lengths (Keivin Burns), 419.
Mercury Lamp, A Low Voltage, Low Wattage (E. L. Harrington), 689.
Micrometer, A New Ocular (Hermann Kellner), 889.
Microphotometer, Registering (Edison Pettit, and Seth B. Nicholson), 76, 187.
Microscope Interferometer, (F. Twyman), 635.
Mirror, Note on Lloyd's (J. B. Green), 299.
MOFFITT, G. W.: The Measurement and Specification of Optical Characteristics in Projector Performance, 65.
MOFFITT, G. W.: An Instrument for the Testing of Prisms, 831.
MOHLER, F. L., PAUL D. FOOTE, A. E. RUARK, and C. C. KIESS: The Excitation of Spark Spectra of Rubidium and Caesium in a Low Voltage Arc, 62.
MOHLER, F. L., PAUL D. FOOTE and A. E. RUARK: The D_2 Zeeman Pattern for Resonance Radiation, 415.
MOHLER, F. L. and ARTHUR E. RUARK: Stages in the Excitation of the Spectra of Thallium, 819.
MORSE, ROBERT V.: Grinding Non-Spherical Lenses, 705.
Munsell and Fechner Scales of Luminous Sensation (Elliot Q. Adams), 70.

N

NICHOLSON, SETH B. and EDISON PETTIT: The Registering Microphotometer of the Mount Wilson Observatory, 76, 187.
Notes, 126, 150, 174, 178, 194, 204, 212, 218, 230, 262, 280, 286, 326, 354, 362, 373, 374, 383, 384, 388, 417, 418, 437, 438, 454, 494, 500, 508, 516, 528, 535, 536, 580, 626, 634, 656, 660, 664, 676, 677, 687, 692, 704, 718, 818, 829, 830, 852, 856, 872, 887, 891, 899, 900, 906, 912, 981, 982, 995, 1009, 1010, 1013, 1016, 1025, 1066, 1067, 1068, 1078, 1121, 1154, 1159, 1160, 1174, 1210.
Notices, 144, 209, 304, 542, 581.
NUTTING, P. G.: The Design of Large Incandescent Lamps, 399.
NUTTING, P. G.: Light Distribution in Optical Systems, 407.

O

Ocular Micrometer (Hermann Kellner), 889.
Ohmmeters, Crossed-Coil (Arthur Steinert), 281.
OLSHO, SIDNEY L.: Fixed Points for Refracting and Spectacle Fitting, 355.
Optical Characteristics in Projector Performance (G. W. Moffitt), 65.
Optical Instruments, Exhibit of, 82.
Optical Pyrometer, Improved Forms of the Disappearing Filament (C. O. Fairchild and W. H. Hoover), 77, 543.
Optical Society of America, Proceedings of the Seventh Annual Meeting of, 59.
Optical Society of America, Rochester Section of, 1025.
Optical Systems, Light Distribution in (P. G. Nutting), 407.
Optical Systems, Longitudinal Aberration in Aspherical (Irwin Roman), 861.
Optics of the Atmosphere (W. J. Humphreys), 61.

P

PALMER, JR., FREDERIC, Some Properties of Atoms and Electrons as Measured by Students, 873.
Parabolic Surfaces (W. B. Rayton), 197.
Parallel Plates, Reflection and Transmission by (Frank Benford), 1017.
Pendulum, Period of Bifilar (H. S. Uhler), 263.
Perception of Depth, An Apparatus for Testing the (Hermann Kellner), 857.
PETTIT, EDISON and SETH B. NICHOLSON: The Registering Microphotometer of the Mount Wilson Observatory, 76, 187.
PFUND, A. H.: The Relation Between The Hiding-Power and Reflection Coefficients of White Pigments and Paints, 78.
Photic Orientation in Organisms (S. O. Mast), 61.
Photoelectric Transmissometry (K. S. Gibson), 693.
Photographic Densities, An Adaptation of the Thalofide Cell to the Measurement of (Arthur L. Schoen), 80, 483.
Photographic Densities, An Instrument for the Measurement of (C. B. Bazzoni, R. W. Duncan and W. S. Mathews), 1003.
Photographic Densities, An Instrument for the Measurement of (Loyd A. Jones), 80, 231.
Photographic Densities (See Densitometer).

Photographic Emulsion, Energy Exchanges in the Formation of the Latent Image of a (S. E. Sheppard and E. P. Wightman), 58.
Photographic Films During Development, The Shrinkage of (H. L. Curtis), 79, 275.
Photographic Exposure, on the Relation Between Time and Intensity in (Loyd A. Jones and Emery Huse), 80, 1079.
Photographic Materials, Spectral Energy Sensitivity of (Loyd A. Jones and A. L. Schoen), 81, 213.
Photographic Sensitometer, (Loyd A. Jones), 80, 305.
Photometer, A Chart of the Flicker (Herbert E. Ives), 363.
Photometer for Illuminants, A Color Match (Herbert E. Ives), 75, 243.
Photometric Equipment for Integrating Spheres (A. H. Taylor), 76.
Photometric Integrator, A Hemispherical (Frank Benford), 76.
Photometry, Comparative Studies of Equality of Brightness and Flicker (Gertrude Rand), 72.
Photometry, The Theory of Flicker (C. E. Ferree), 71.
Photometry and Colorimetry of Daylight and Incandescant Illuminants by the Method of Rotatory Dispersion (Irwin G. Priest), 75, 1175.
Photomicrography with Ultraviolet Light, An Automatic Focusing Device for (W. T. Bovie and C. E. Barr), 1167.
Pigments and Paints, Hiding Power and Reflection Coefficients of (A. H. Pfund), 78.
Polarized Light, A Study of the Phenomena Exhibited by Transparent Inactive Crystal Plates in Convergent (F. E. Wright), 779.
Polarized Light, Projection of (Lloyd W. Taylor), 221.
PORITSKY, A. and ENOCH KARRER: An Improved Slide Wire Resistance, 277.
Potentiometers, Multiple Range (Leo Behr), 665.
Potentiometers for Measuring Hydrogen-ion Concentrations, Methods of Making Temperature Corrections in Direct Reading (W. T. Bovie), 1235.
PRIEST, IRWIN G.: Progress on the Determination of Normal Gray Light, 72.
PRIEST, IRWIN G.: Preliminary Data on the Color of Daylight at Washington, 78.
PRIEST, IRWIN G.: The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion, 75, 1175.
PRIEST, IRWIN G., and Casper L. Cottrell: The Effect of Various Conditions Upon the Determination of the Normal Stimulus of Gray, 73.
Prisms, An Instrument for the Testing of (G. W. Moffitt), 831.
Proceedings of the Seventh Annual Meeting of the Optical Society of America, 59.
Projector Performance (G. W. Moffitt), 65.
Psychophysiological Optics, Some Recent Contributions to (L. T. Troland), 65.
Pump, Mercury Vapor (L. T. Jones), 537.
Pyrheliometer, A New Form of Thermo-electric Recording (Herbert H. Kimball and Hermann E. Hobbs), 707.
Pyrometer, Disappearance of the Filament and Diffraction Effects in Improved Forms of An Optical (C. O. Fairchild and W. H. Hoover), 77, 543.

R

Radiant Energy, The Visibility of (K. S. Gibson and E. P. Tyndall), 68.
Radiant Energy. Transformation into Electric Current in Molybdenite (W. W. Coblentz), 63.
Radioactivity, An Instrument for Showing Some of the Phenomena of (J. E. Underwood), 1155.
Radioactivity. A Projection Electroscope for Measurements in (Howard M. Elsey), 385.
RANDALL, H. M.: Report on the Fine Structure of Near Infrared Absorption Bands, 45.
RAND, GERTRUDE: Comparative Studies of Equality of Brightness and Flicker Photometry with Spectral Reference to the Lag of Visual Sensation, 72.
Ray Tracing (C. W. Woodworth), 673.
RAYTON, W. B.: The Practical Application of Parabolic Surfaces in Lens Construction, 65, 197.
Reciprocity Law, A Special Sensitometer for the Study of the Photographic (Loyd A. Jones), 80, 305.
Reflecting Powers of Tellurium (L. P. Sieg), 147.
Reflection and Transmission by Parallel Plates (Frank Benford), 1017.
Reflection Coefficients and Hiding-Power of Pigments and Paints (A. H. Pfund), 78.
Reflection of Light from a Matt Surface (Eugene M. Berry), 627.
Reflection of Light from a Sphere, (Eugene M. Berry), 679.
Reflection of Ultraviolet by Flowers, (F. K. Richtmyer), 62, 151.
Reflex Nature of Color Contrast (Frank Allen), 913.
Reflex Visual Sensations (Frank Allen), 583.
Refracting and Spectacle Fitting, Fixed Points for (Sidney L. Olsho), 355.
Refraction of a Thin Pencil by any Refracting Surface: Generalized Meridional and Sagittal Formulas, (E. D. Roe, Jr.), 64.
Report of Business Manager, 778.
Report of Secretary, 144, 209, 304, 542, 581.
Report of Treasurer, 777.

Resistance, An Improved Slide Wire (Enoch Karrer and A. Poritsky), 277.
Resonance Radiation, The D_2 Zeeman Pattern for (Paul D. Foote, A. E. Ruark and F. L. Mohler), 415.
Retinal Response, (E. L. Chaffee and W. T. Bovie), 61.
Retinal Response (E. L. Chaffee, W. T. Bovie, and Alice Hampson), 1.
Reviews
BOLTON, L.: An Introduction to the Theory of Relativity, 953.
DENDY, ARTHUR, (Editor): Problems of Modern Science, 1029.
FOOTE, PAUL D. and F. L. MOHLER: The Origin of Spectra, 207.
GREEN, JOHN, and A. E. EWING: Optotypes, Test Letters and Pictographs for Measuring the Acuteness of Vision, 1042.
International Committee Appointed by the 7th Congress of Applied Chemistry: Annual Tables of Constants and Numerical Data, 903.
JONES, H. SPENCER: General Astronomy, 672.
KONIG, A.: Die Fernrohre und Entfernungsmesser, 669.
LORING, F. H.: Atomic Theories, 1030.
LUCKIESH, M.: Visual Illusions, Their Causes, Characteristics and Applications, 142.
LUCKIESH, M.: Ultraviolet Radiation, 688.
PASCHEN, F. and R. GOTZE: Seriengesetze der Linienspektren, 1113.
PEDDIE, W.: Colour Vision, 1251.
REICHE, FRITZ: The Quantum Theory, 320.
SHAW, JAMES BYRNIE: Vector Calculus, 892.
SILBERSTEIN, L.: Synopsis of Applicable Mathematics, 888.
SMITH, ALPHEUS W.: The Elements of Applied Physics, 776.
STEINMETZ, CHARLES P.: Relativity in Space, 971.
TROLAND, L. T.: The Present status of Visual Science, 775.
WEBSTER, D. L., H. W. FARWELL and E. R. DREW: General Physics for Colleges, 954.
Rheostat with Constant Contacts, A Continuously Variable (Paul Kirkpatrick), 195.
RICHTMYER, F. K.: The Reflection of Ultraviolet by Flowers, 62, 151.
ROBERTS, J. M.: Standardization and Limitation of Types and Sizes of Laboratory Apparatus, 973.
ROBERTSON, JOHN K.: Determination of the Schwarzschild Constant, 996.
ROBERTSON, JOHN K.: Intensity Measurements of Spectral Lines, 983.
ROE, JR., E. D.: Refraction of a Thin Pencil by any Refracting Surface: Generalized Meridional and Sagittal Formulas, 64.
ROMAN, IRWIN: Longitudinal Aberration in Aspherical Optical Systems, 861.
Rotatory Dispersion Applied to Colorimetry and Photometry (Irwin G. Priest), 75, 1175.
RUARK, A. E., C. C. KIESS, F. L. MOHLER, and PAUL D. FOOTE: The Excitation of Spark Spectra of Rubidium and Caesium in a Low Voltage Arc, 62.
RUARK, ARTHUR E. and F. L. MOHLER: Stages in the Excitation of the Spectra of Thallium, 819.
RUARK, A. E., F. L. MOHLER and PAUL D. FOOTE: The D_2 Zeeman Pattern for Resonance Radiation, 415.

S

SCHLINK, FREDERICK J.: Life Testing of Mechanical Instruments, 1031.
SCHOEN, A. L.: An Adaptation of the Thalofide Cell to the Measurement of Photographic Densities, 80, 483.
SCHOEN, A. L. and LOYD A. JONES: Preliminary Note on the Spectral Energy Sensitivity of Photographic Materials, 81, 213.
Schwarzschild Constant, Determination of the (John K. Robertson), 996.
Screening Doublets in X-Ray Spectra (A. Sommerfeld), 503.
Screws, Simple Equipment for Detecting the Errors of (Wilmer Souder), 64.
Searchlight Beam, Diffusion of Light from (Enoch Karrer and U. M. Smith), 1211.
Sector Disk in X-Ray Measurements (J. A. Becker and F. W. Warburton), 127.
Sectored Disc, A Variable Aperture Rotating (Herbert E. Ives), 683.
Sectored Disk, Variable and Electrically Readable While in Rotation (Enoch Karrer), 893.
Sensitometer for the Study of the Photographic Reciprocity Law (Loyd A. Jones), 80, 305.
SHEARD, CHARLES: The Horopter, Cyclophoria and Apparent Vertical Meridian (Preliminary Report), 81.
SHEPPARD, S. E., and E. P. WIGHTMAN: Note on the Energy Exchanges in the Formation of the Latent Image of a Photographic Emulsion, 58.
SIEG, L. P.: Note on the Principal Reflecting Powers of Tellurium, 147.
SILBERSTEIN, L.: Aspherical Lens Systems, 82.
SINDEN, R. H.: On the Luminosity Ratios of the Spectral Complementaries, and the Subjective Saturation of the Spectrum, 69.

SINDEN, R. H.: Studies Based on the Spectral Complementaries I. Subjective Saturation of Spectral Hues, II. Individual Variations of the Normal Color-Sense, 1123.
Slide Wire Resistance (Enoch Karrer and A. Poritsky), 277.
SLIGH, JR., T. S.: Note on Platinum Resistance Thermometer Construction, 203.
Slits, Effect of the Material Composing the Sides of Deep Slits on the Intrinsic Intensity of Light Transmitted Through the (Clarence R. Smith), 1069.
SMITH, CLARENCE R.: The Effect of the Material Composing the Sides of Deep Slits on the Intrinsic Intensity of Light Transmitted Through the Slits, 1069.
SMITH, U. M. and ENOCH KARRER: Diffusion of Light from a Searchlight Beam, 1211.
Solar Radiation Work, Apparatus and Results of (C. G. Abbot), 61.
SOMMERFELD, A.: Note on Brewster's Law, 501.
SOMMERFELD, A.: Regularities in the Screening Constants of Rontgen Spectra, 503.
SOMMERFELD, A.: The Model of the Neutral Helium Atom, 509.
SOMMERFELD, A.: Coupled Oscillations of a Helical Spring, 529.
SOUDER, WILMER: Simple Equipment for Detecting the Errors of Screws, 64.
Sound Waves, A Simple Device for Recording (C. J. Lapp), 661.
SOUTHALL, JAMES P. C.: Abolition of Astigmatism of Oblique Bundles in Case of a Single Lens, 219.
Spark Gap, Air-Cooled (W. T. Bovie), 1027.
Spectacle Fitting, Fixed Points for Refracting and (Sidney L. Olsho), 355.
Spectral Complementaries (R. H. Sinden), 69, 1123.
Spectral Energy Distribution, Tables and Graphs (M. Katherine Frehafer), 74.
Spectral Energy Sensitivity of Photographic Materials (Loyd A. Jones and A. L. Schoen), 81, 213.
Spectral Transmission, Direct-Reading Photoelectric Measurement of (K. S. Gibson), 693.
Spectra of Thallium, Stages in the Excitation of the (F. L. Mohler and Arthur E. Ruark), 819.
Spectra, Spark, of Rubidium and Caesium in a Low Voltage Arc (F. L. Mohler, Paul D. Foote, A. E. Ruark and C. C. Kiess), 62.
Spectrograph, A Simple Prism (E. O. Hulburt), 205.
Spectrograph, Vacuum Grating (Theodore Lyman), 495.
Spectrometer Crystals, A Device for Rotating (Paul Kirkpatrick), 539.
Spectrometer Table (B. J. Spence), 853.
Spectroradiometry (W. W. Coblentz), 439.
Spectroscope, A Method of Increasing the Dispersion of a Constant Deviation (J. F. T. Young), 405.
SPENCE, B. J.: A Spectrometer Table, 853.
SPENCE, B. J. and C. HOLLEY: The Infrared Absorption of Hydrogen Chloride in the Region 3.5μ and at 200°K, 169.
Stand, An Improved Burette and Filter (Clifford D. Carpenter), 671.
STANNARD, W. H. and PAUL E. KLOPSTEG: An Improved Method for the Contro! of Temperature and Other Physical Quantities, 179.
STEINERT, ARTHUR: The Performance of Crossed-Coil Systems in Direct Reading Ohmmeters, 281.
Stellar and Planetary Radiation, Thermocouple Measurements of (W. W. Coblentz), 61.
STUHLMAN, JR., OTTO: Design of a Kundt's Tube, 905.

T

TAYLOR, A. H.: Improvements in Photometric Equipment for Integrating Spheres, 76.
TAYLOR, LLOYD W.: The Projection of Polarized Light Phenomena, 221.
Temperature Control, An Improved Method for (Paul E. Klopsteg and W. H. Stannard), 179.
Thalofide Cell for Measurement of Photographic Densities (Arthur L. Schoen), 80, 483.
Thermoelectric Power of Tin at the Melting Point, the Change in (T. R. Harrison and Paul D. Foote), 389.
Thermoelectric Pyrheliometer (Herbert H. Kimball and Hermann E. Hobbs), 707.
Thermometer Construction, Note on Platinum Resistance (T. S. Sligh, Jr.), 203.
Transmissometry, Photoelectric (K. S. Gibson), 693.
TROWBRIDGE, A. and K. T. COMPTON: Note on Brauer's Theory of the Function of the Cathode in Electric Arcs, 321.
TROLAND, L. T.: Some Recent Contributions to Psychophysiological Optics, 65.
TROLAND, L. T. and C. H. LANGFORD: Further Studies of the Abney Effect, 70.
TWYMAN, F.: The Hilger Microscope Interferometer, 635.
TYNDALL, E. P. and K. S. GIBSON: The Visibility of Radiant Energy, 68.

U

UHLER, H. S.: Period of the Bifilar Pendulum for Large Amplitudes, 263.
UHLER, H. S.: Method of Least Squares and Curve Fitting, 1043.
Ultraviolet by Flowers, the Reflection of (F. K. Richtmyer), 62, 151.

Ultraviolet Focusing, Device for (W. T. Bovie and C. E. Barr), 1167.
UNDERWOOD, J. E.: An Instrument for Showing Some of the Phenomena of Radioactivity, 1155.

V

Viscosimeter, The Improved MacMichael (Winslow H. Herschel), 335.
Visibility of Radiant Energy (K. S. Gibson and E. P. Tyndall), 68.
Vision of School Children, Tests for the Acuteness of (J. M. McCallie), 70.
Visual Sensations and Color Contrast (Frank Allen), 913.
Visual Sensations, Lag of, with Reference to Photometry (Gertrude Rand), 72.
Visual Sensations, Reflex (Frank Allen), 61, 583.
Voltmeter-Ammeter, A Projection (Howard M. Elsey), 403.

W

WARBURTON, F. W. and J. A. BECKER: The Use of a Sector Disk in X-Ray Measurements, 127.
WARNER, JOHN A. C.: An Optical Altitude Indicator for Night Landing, 225.
Wave Lengths, On the Measurement of Standard (Keivin Burns), 419.
Wave Lengths, Summary of Investigations in the Infrared Spectrum of Long (W. Weniger), 517.
WEINHART, H. W. and J. B. JOHNSON: Apparatus for Admitting Gas Into A Vacuum, 175.
WENIGER, W.: Summary of Investigations in the Infrared Spectrum of Long Wave Lengths, 517.
WIGHTMAN, E. P., and S. E. SHEPPARD: Note on the Energy Exchanges in the Formation of the Latent Image of a Photographic Emulsion, 58.
WILLIAMS, S. R.: An Extensometer, Amplifier, 1011.
WOODWORTH, C. W.: Methods for Machine Calculation of Rays Through A Lens System, 673.
WRIGHT, F. E.: The Formation of Interference Figures. A Study of the Phenomena Exhibited by Transparent Inactive Crystal Plates in Convergent Polarized Light, 779.

X

X-Ray Measurements, the Use of a Sector Disk in (J. A. Becker and F. W. Warburton), 127.
X-Rays, A New Method of Crystal Analysis and the Reflection of Characteristic (George L. Clark and William Duane), 455.
X-Rays, Extraordinary Diffraction of (L. W. McKeehan), 63.
X-Ray Spectra, Regularities in the Screening Constants of (A. Sommerfeld), 503.

Y

YOUNG, J. F. T.: A Method of Increasing the Dispersion of A Constant Deviation Spectroscope, 405.

Z

Zeeman Pattern for Resonance Radiation (Paul D. Foote, A. E. Ruark and F. L. Mohler), 415.

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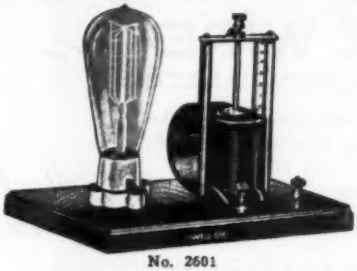

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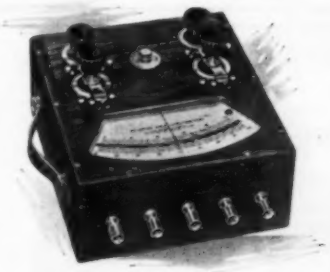

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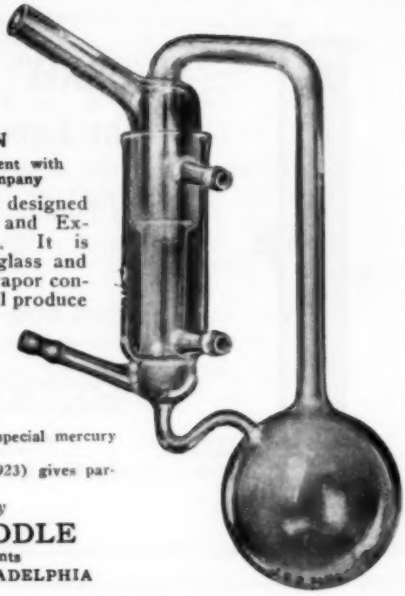

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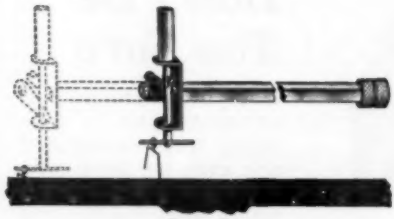

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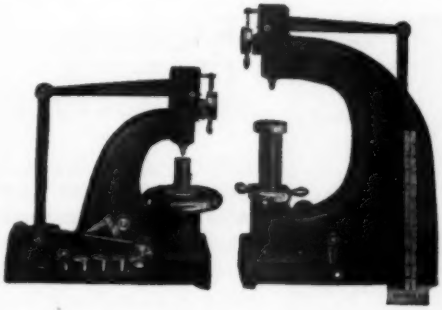

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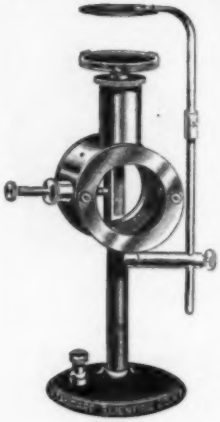

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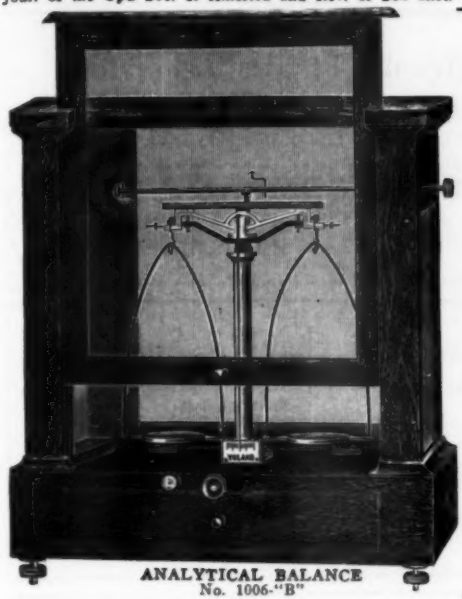

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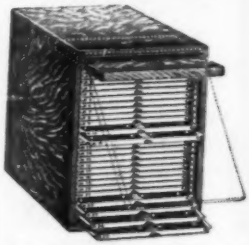

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